Coursework #1

Deadline: Wednesday, 25 February 2009, 15:00

Question 1 (10 marks)

A social welfare function is said to satisfy the axiom of *non-imposition* (NI) if any social preference ordering is achievable by *some* profile of individual preference orderings:

$$(\forall P \in \mathcal{P})(\exists \boldsymbol{P'} \in \mathcal{P}^n)(\forall x, y \in A)[xPy \leftrightarrow xP'y]$$

In other words, a social welfare function satisfying (NI) does not impose any restrictions that would *a priori* exclude a particular social preference ordering.

(a) Show that the weak Pareto condition (WP) implies (NI).

(b) Show that Arrow's Theorem breaks down if we replace (WP) by (NI).

(Adapted from A.D. Taylor, *Social Choice and the Mathematics of Manipulation*, Cambridge University Press, 2005.)

Question 2 (10 marks)

A *voting correspondence* is a function mapping a set of linear orders (the voter preferences) over the set of candidates to a nonempty subset of the set of candidates (the winners).

(a) Choose three of the voting correspondences introduced in class and check whether they satisfy the Pareto principle (that is, give a proof in the affirmative case, and a counterexample otherwise).

(b) Suggest a reasonable definition of *independence of irrelevant alternatives* (IIA) for voting correspondences and justify your proposal.

(c) For the same three voting correspondences as those chosen in (a), check whether they satisfy your formulation of (IIA).

Question 3 (10 marks)

In analogy to the definition of Condorcet winners, a *Condorcet loser* is a candidate that would lose against any other candidate in a pairwise contest.

(a) Give an example that shows that the plurality rule *can* elect a Condorcet loser.

(b) Prove that the Borda rule *never* elects a Condorcet loser.

Remark: It is in fact possible to show that the Borda rule is the *only* positional scoring rule (with a strictly descending scoring vector) that satisfies this property.