Coursework #3

Deadline: Friday, 4 April 2008, 3:00pm

Question 1 (10 marks)

Suppose three towns, A, B and C, are located in the plane $\mathbb{R}^2$. We have to decide where to build a new hospital H. Any point in the plane is feasible. The *dis*utility of a town is the distance of that town to H.

(a) Show that the Pareto optimal locations for H are precisely those that are lying within the triangle ΔABC.

(b) Show that we have an equality-efficiency dilemma iff that triangle is obtuse angled (that is, iff it has an angle of more than 90 degrees).

(c) Give a geometric characterisation of the optimum of the egalitarian CUF in case the triangle is obtuse angled.

(Adapted from H. Moulin, *Axioms of Cooperative Decision Making*, CUP, 1988.)

Question 2 (10 marks)

Suppose there are n agents located anywhere on the interval $[0, 1]$. We have to decide where to build an amusement park A, also anywhere on the same interval. The *dis*utility of an agent is its distance to A.

(a) What is the solution selected by the egalitarian CUF?

(b) What is the solution selected by the elitist (n-rank dictator) CUF?

(c) For arbitrary $k \leq n$, give a general procedure to compute the solution that would be optimal with respect to the k-rank dictator CUF.

Question 3 (10 marks)

What is the computational complexity of (the decision variant of) the problem of finding an allocation of resources that maximises elitist social welfare?

(a) First state your answer (and your proof) with respect to the explicit form of representing utility functions (where the size of the representation of a utility function is taken to be proportional to the number of bundles to which it assigns a non-zero value).

(b) Then repeat the same exercise, this time assuming that utility functions are expressed using the language of weighted propositional formulas (without restrictions).