EQUITABLE ALLOCATIONS OF EARTH OBSERVING SATELLITE RESOURCES

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context

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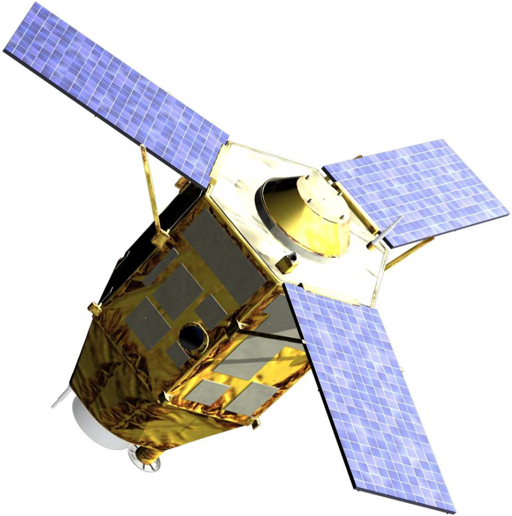
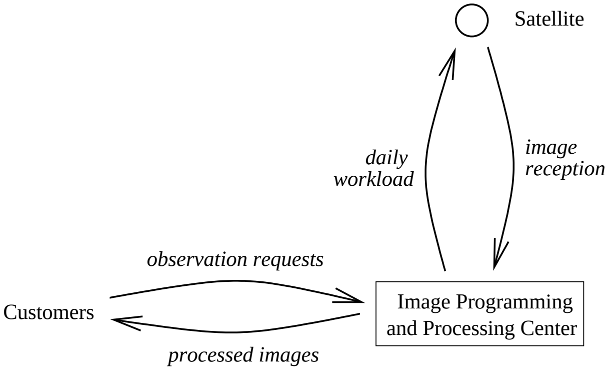


Earth Observing Satellite (EOS) : how does it work ?

The mission of Earth Observing Satellites :
to acquire *images*, in response to *requests* from customers.



DEMO PLEIADES

Equitable allocation for EOS : the problem (informal)

The satellite (or a constellation of satellites)
is *co-funded by several agents* ...
... and then *exploited in common*.

ex : PLEIADES $\rightarrow$ France/Italy, civil/defense

The common exploitation must be

- *efficient* :
  the satellite(s) must not be under-exploited
- *equitable* :
  for each agent,
  its "return on investment" should be proportional
  to its financial contribution.

our work :

→ to define efficient and equitable allocation procedures for Earth Observing Satellites, in different contexts.

1. set the principles
2. design methods/algorithms following the principles.

An image request is characterized by :

- the requesting *agent*
- its location, size, ...
- its imaging *constraints* (ex : mono or stereo, shooting angle ...) and validity window (ex : from next June 15 to August 30)
- its *weight*
  (measure of its importance $\rightarrow$ expression of *preferences*)

Generally, all requested images cannot be processed,
due to *conflicts* between them
(respect of physical and imaging constraints, minimum transition time between images ...).

The daily (repetitive) problem :

- select, among the set of valid image requests,
  a subset of images to be taken the next day.
  (subset of selected images = an *allocation of images to agents*).
- the allocation must be *admissible* (no conflicts)
- the allocation should be *efficient* and *equitable*,
  as much as possible.

equitable allocations : two main approaches

1. *decentralized game* :
   Free interactions between agents, obeying a rule.
   Design a rule such that negotiations between agents converge towards an equitable allocation
   $\longrightarrow$ too long and difficult, often lacks efficiency.
2. *centralized arbitration procedure* :
   Justice given by a fair and impartial procedure (arbitrator)
   $\longrightarrow$ more appropriate (automatic, confidential, efficient).

A simple model for the fair allocation problem

- $N = \{1, \cdots, n\}$: *agents*
- $\mathcal{O}$: *indivisible objects* (images)
- $\Delta_i \subseteq \mathcal{O}$: *demands* of agent i
- $\mathbf{x} = \langle x_1, \cdots, x_n \rangle$: an *allocation*
  $x_i \subseteq \Delta_i$: the *share* of agent i in $\mathbf{x}$
- Adm : set of *admissible allocations*
- $\mathbf{q} = \langle q_1, \cdots, q_n \rangle$ with $0 < q_i < 1$ and $\sum_i q_i = 1$
  q_i : the *quota* of agent i (entitlement).

- $w_i(o) \in \mathbb{R}^{+*}$: *weight* given by agent i to object o
  weights are set freely by agents
- $u_i(\mathbf{x}) \in \mathbb{R}^{+}$: *individual utility* of $\mathbf{x}$ for i,
  measure of individual satisfaction
- $uc(\mathbf{x}) \in \mathbb{R}^{+}$: *collective utility* of $\mathbf{x}$,
  measure of collective (or arbitrator) satisfaction

Each agent i wants to maximize his individual utility $u_i(\mathbf{x})$.

The society (or the benevolent arbitrator) will choose an allocation maximizing the collective utility $uc(\mathbf{x})$.

How to define $u_i(\mathbf{x})$ and $uc(\mathbf{x})$?
$\rightarrow$ from $\mathbf{x}$, the agents demands, and the weights of objects.

utility definitions : two phases agregation

$$\left.\begin{array}{c}(\Delta_1, \mathsf{x}) \mapsto u_1(\mathsf{x}) \\ \dots \\ (\Delta_n, \mathsf{x}) \mapsto u_n(\mathsf{x})\end{array}\right\} \mapsto uc(\mathsf{x})$$

phase 1 : individual utility

The most simple approach :

- the satisfaction of an agent does not depend on other agents satisfactions
- weights are *additive* (full compensation) (agents are indifferent to get 2 objets of weight 1 or 1 object of weigth 2)

$\longrightarrow$

$$u_i(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{o \in x_i} w_i(o)$$

normalization of individual utilities

To be able to *compare* the satisfaction of agents,
we need to express individual utilities on a *common scale*.

Maximal individual utility :

$$\widehat{u}_i \stackrel{\text{def}}{=} \max_{\mathbf{x} \in Adm} u_i(\mathbf{x})$$

→ *Normalized individual utility* :

$$u'_i(\mathbf{x}) \stackrel{\text{def}}{=} \frac{u_i(\mathbf{x})}{\widehat{u}_i}$$

(Kalai-Smorodinsky)

phase 2 : collective utility

$$uc(\mathbf{x}) = g(\langle u'_1(\mathbf{x}), \cdots, u'_n(\mathbf{x})\rangle, \mathbf{q})$$

Desirable properties :

- strict monotonicity (Pareto-efficiency)
  $uc(\mathbf{x})$ should not decrease when $u_i(\mathbf{x})$ increases
- equity
  $\rightarrow$ symetry (anonymicity)
  $\rightarrow$ «fair share», «inequality reduction (Pigou-Dalton)», ... ?

Many many possibilities ...

Different approaches for the collective utility function

Which collective utility function *uc* ?

«Ethical» choices :

- egalitarianism [Rawls]
- utilitarianism [Keeney, Harsani ...]
- compromises
- partial orderings.

pure egalitarianism

Probably the simplest and most appropriate method among those investigated :
choose an allocation **x** which maximizes

$$uc(\mathbf{x}) \stackrel{\text{def}}{=} \min_i \frac{u'_i(\mathbf{x})}{q_i}$$

→ tend to *maximize* the $u'_i(\mathbf{x})$ and make them *proportional* to q_i.

Needs a small improvement to get full Pareto-efficiency :
the leximin *preordering*.

pure utilitarianism

with equal quotas : $uc(\mathsf{x}) = \sum_i u'_i(\mathsf{x})$

(normalization and symetry are minimal equity requirements)

with unequal quotas : $uc(\mathsf{x}) = \sum_i q_i \cdot u'_i(\mathsf{x})$

The arbitrator is indifferent between
giving $\Delta u'_i$ to i or giving $\Delta u'_j$ to j, if $q_i \cdot \Delta u'_i = q_j \cdot \Delta u'_j$,
not considering whether i is already richer or poorer than j.

$\rightarrow$ *in this approach, equity is not a strong concern.*

compromises : OWA

Ordered Weighted Averaging (OWA) operators [Yager 88]

$$u'(\mathsf{x}) \stackrel{\text{def}}{=} \langle u'_1(\mathsf{x}), u'_2(\mathsf{x}), \ldots, u'_n(\mathsf{x})\rangle$$

$$u^\star(\mathsf{x}) \stackrel{\text{def}}{=} \langle u^\star_1(\mathsf{x}), u^\star_2(\mathsf{x}), \ldots, u^\star_n(\mathsf{x})\rangle$$

the same as $u'(\mathsf{x})$ but sorted increasing. Then

$$uc(\mathsf{x}) \stackrel{\text{def}}{=} \sum_i \alpha^{i-1} \cdot u^\star_i(\mathsf{x}),\ \textit{with}\ \alpha \in]0,1].$$

- $\alpha = 1 \rightarrow$ pure utilitarianism
- α small enough $\rightarrow$ egalitarianism (leximin preordering).

compromises : SE

«Sum of Exponents» operators [see Moulin 1988 or 2003]
Additive family.

$$uc_{(p)}(\mathbf{x}) \stackrel{\text{def}}{=} \sum_i g_{(p)}(u_i'(\mathbf{x})), \quad p \leq 1$$

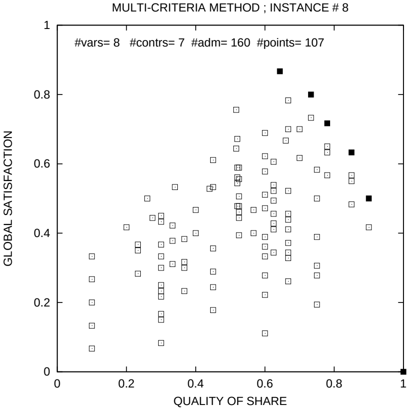
$$g_{(p)}(u) \stackrel{\text{def}}{=} \mathrm{sgn}(p) \cdot u^p \ , \ p \neq 0$$

$\mathrm{sgn}(p) \stackrel{\text{def}}{=} 1$ if $p > 0$, $\mathrm{sgn}(p) \stackrel{\text{def}}{=} -1$ if $p < 0$

$$g_{(0)}(u) \stackrel{\text{def}}{=} \log u \quad (\textit{Nash})$$

- $p = 1$: pure utilitarianism
- $p \rightarrow -\infty$: egalitarianism (leximin preordering).

a quite different approach : two collective criteria



Two criteria :

1. global satisfaction : $\dfrac{1}{n}\sum_i u_i'(\mathbf{x})$
2. quality of share : inequality indice (such as Gini)

an advanced model : taking into account complex demands

The presented model : *simple* demands.

But sometimes we need more complex demands,
such as (real-world examples) :

- stereoscopic images (reinforcement effect)
- images from different revolutions (weakening effect)

→ *compact representation langage for complex demands*
(Sylvain and Jerome talks)

summary

1. A *real-word* problem : equitable allocation of satellite resources among several agents.
2. A formal *model*, for the allocation of indivisible objects between some agents, based upon two levels of *utilities*.
3. Several *collective utility functions* have been considered, qualifying *efficient* and *equitable* allocations,
   with different «ethical» perceptions.

The equitable allocation problem is strongly linked to

- (compact) expression of preferences
  (more on that with Jerôme and Sylvain)
- combinational auctions
- cooperative microeconomics.

open or still ill-solved problems

- collective utility functions (CUF) and
  - *entitlements* for compromises (OWA, SE)
  - *entitlements* as maximum amount of resource consumptions
  - *strategyproof* preference declarations
- taking advantage of the *repetitive* nature of the problem (temporal compensations)
- other characterizations of *equity* in this context
- algorithmics : for optimizing the CUF
  - quick/approximate algorithms for very large instances
  - heuristics for selecting objects.

Cardinal characterizations of equity

(Ordinal ones, such as envy-freeness,
are considered by Jerôme and Sylvain)

an equity test : the fair share

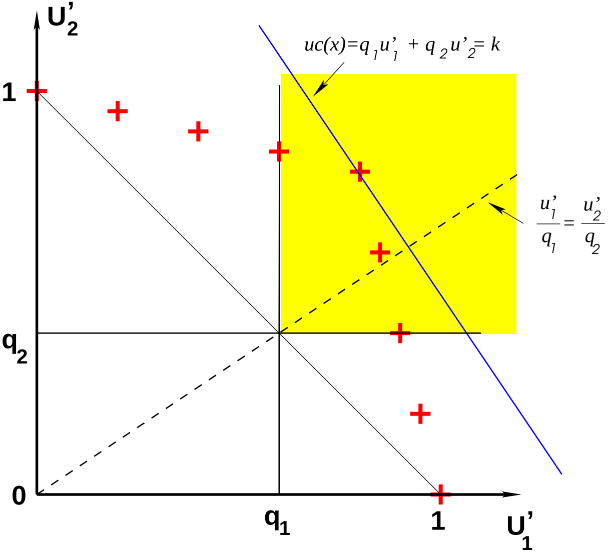
Agent i receives a *fair share* iff

$$u_i(\mathbf{x}) \geq \widehat{u}_i \cdot q_i$$

which is equivalent to

$$q_i \leq u_i'(\mathbf{x})$$

Note : doesn't need intercomparability of individual utilities.



U'2
uc(x)=q1u'1 + q2u'2= k
1
u'1/q1 = u'2/q2
q2
0
q1
1
U'1

inequality reduction : the Pigou-Dalton property

(see [Moulin 1988 or 2003])

Aversion for pure inequality.

An *inequality reduction* from $\mathbf{x}$ to $\mathbf{y}$ occurs iff :

- $u'_1(\mathbf{y}) + u'_2(\mathbf{y}) = u'_1(\mathbf{x}) + u'_2(\mathbf{x})$
  (sum of individual utilities are preserved)
- $u'_1(\mathbf{x}) < u'_1(\mathbf{y}) < u'_2(\mathbf{y}) < u'_2(\mathbf{x})$
  or
  $u'_1(\mathbf{x}) < u'_2(\mathbf{y}) < u'_1(\mathbf{y}) < u'_2(\mathbf{x})$.

The Pigou-Dalton property requires that, if there is an inequality reduction from $\mathbf{x}$ to $\mathbf{y}$, then *uc* does not decrease.

formal properties of utility functions (see [Moulin 03])

When considering equity, the following properties are desirable :

- monotonicity (Pareto-efficiency)
- symetry (anonymicity)
- independance of unconcerned agents (IUA) (separability)
- inequality reduction (Pigou-Dalton property)
- independance of common utility scale (ICS).

SE operators obey all these properties.

leximin definition [Aspremont and Gevers 1977]

Let $\mathbf{u}$ be a vector, $\mathbf{u}^\star$ denotes the vector obtained from $\mathbf{u}$ by non decreasing sorting.
Example : $\mathbf{u} = \langle 5, 3, 2, 4, 3 \rangle$, $\mathbf{u}^\star = \langle 2, 3, 3, 4, 5 \rangle$.

- $\mathbf{u}$ and $\mathbf{v}$ are indifferent for the leximin preorder iff $\mathbf{u}^\star = \mathbf{v}^\star$
- $\mathbf{u}$ is prefered to $\mathbf{v}$ for the leximin preorder iff it exists an integer r in $0, \ldots, n-1$ such that

$$u_i^\star = v_i^\star \text{ for } i = 1, \ldots, r, \quad \text{and } u_{r+1}^\star > v_{r+1}^\star$$

formal properties of the leximin [Moulin 03]

The leximin is the only collective utility preorder which satisfies

- inequality reduction (Pigou-Dalton)
- independance of the *common utility pace* (ICP) (ordinality)

utilitarism, egalitarism and equity