# O P-S F N E T - Volume 31, Number 2 - March 15, 2024

The Electronic News Net of the
SIAM Activity Group on Orthogonal Polynomials and Special Functions

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OP-SF Net is distributed to OPSF Activity Group members and non-members alike through the OP-SF Talk listserv.

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Topics:



**Calendar of Events:**

**May 1-3, 2024**
Hypergeometric and Orthogonal Polynomials Event: An international conference
Radboud University Nijmegen, Huygensgebouw, Nijmegen-Oost, The Netherlands
https://www.math.ru.nl/~wzudilin/HOPE-in-May.html

**May 15-17, 2024**
Journées Approximation 2024
Université de Lille, Lille, France

https://indico.math.cnrs.fr/event/11523/

**May 27–31, 2024**
**A**symptotics, **R**andomness, **N**onlinearity, and **O**rthogonality (ARNO 2024)
ARNO 2024 will also be the 2024 Annual Meeting of the PIICQ network,
Leuven, Belgium
https://gsilva.pages.math.cnrs.fr/arno2024/index.html

**June 3–6, 2024**
International Conference on Analysis and Applications
in honor of Roderick S. C. Wong's 80$^{\text{th}}$ birthday,
City University of Hong Kong, Hong Kong
https://www.cityu.edu.hk/rcms/icaa2024/index.html

**June 6–9, 2024**
The Legacy of Ramanujan 2024
Celebrating the 85$^{\text{th}}$ birthdays of George Andrews & Bruce Berndt,
Penn State University, State College, Pennsylvania, USA
https://sites.psu.edu/ramanujan/

**June 24–28, 2024**
17$^{\text{th}}$ International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA-17),
Universidad de Granada, Granada, Spain
https://opsfa17.com/

**June 24–28, 2024**
From Classical to Modern Analysis: In memory of Professor José Carlos Petronilho
a Satellite Conference of the 9$^{\text{th}}$ European Mathematical Congress,
Sanlúcar de Barrameda, Cadíz, Spain
https://www.mat.uc.pt/~pgsfop/fcma/index.html

**July 8–12, 2024**
Operator Theory and Approximation 2024
TU Wien, Vienna, Austria
https://haraldworacek.github.io/OTA2024/

**July 15–19, 2024**
9$^{\text{th}}$ European Congress of Mathematics
Seveille, Spain
https://www.ecm2024sevilla.com/

Mini-Symposium on *Special Functions, Orthogonal Polynomials, $q$-Series and Applications*
Organized by Howard Cohl, Robert Maier and Roberto S. Costas-Santos

**September 4–7, 2024**
Approximation Theory and Special Functions (ATSF 2024)
Dedicated to the retirement of George Anastassiou
TOBB Economics and Technology University, Ankara, Türkiye
https://sites.google.com/view/atsf2024

**December 9–13, 2024**
Joint meeting of the NZMS, AustMS and AMS
Auckland, New Zealand

Special Session on *Special Functions, $q$-Series and Beyond*
Organized by Howard Cohl, Ole Warnaar, Nicholas Witte

**May 19–22, 2025**

Constructive Functions 2025
Celebrating Ed Saff's 80th birthday
in conjunction with the 37th Shanks Lecture by Doron Lubinsky
Vanderbilt University, Nashville, Tennessee, USA
https://my.vanderbilt.edu/constructivefunctions2025/

**June 23– 28, 2025**
Combinatorics around the $q$–Onsager algebra
A celebration of the 70th birthday of Paul Terwilliger
Kranjska Gora, Slovenia
https://conferences.famnit.upr.si/event/15/overview

## Topic #1 ——— OP – SF Net 31.2 ——— March 15, 2024

From: Guilherme Silva (silvag@icmc.usp.br)
Subject: Announcement: **ARNO** 2024 meeting May 27–31, 2024, in Leuven, Belgium

Dear colleagues,

This is the second announcement of the conference:
**A**symptotics, **R**andomness, **N**onlinearity, and **O**rthogonality 2024
Leuven, Belgium
May 27–31, 2024

The conference will be the 2024 Annual Meeting of the PIICQ network.

The list of confirmed speakers is on the website of the event, available here. The poster of the conference is attached in figure 1.

**The registration is now open.** We welcome submissions for contributed talks and poster presentations. Young researchers and mathematicians from underrepresented groups are particularly encouraged to apply. For details, please click here. The deadline for registration and submissions is May 1st , 2024.

We greatly appreciate it if you share this announcement with researchers that may be interested in the conference.

With our best wishes,

Guilherme, in the name of the organizing committee:

- Tom Claeys (UC Louvain, Belgium)
- Maurice Duits (KTH Stockholm, Sweden)
- Manuela Girotti (Emory University, USA)
- Leslie Molag (Carlos III University of Madrid, Spain)
- Guilherme Silva (Universidade de São Paulo, Brazil)
- Walter Van Assche (KU Leuven, Belgium)

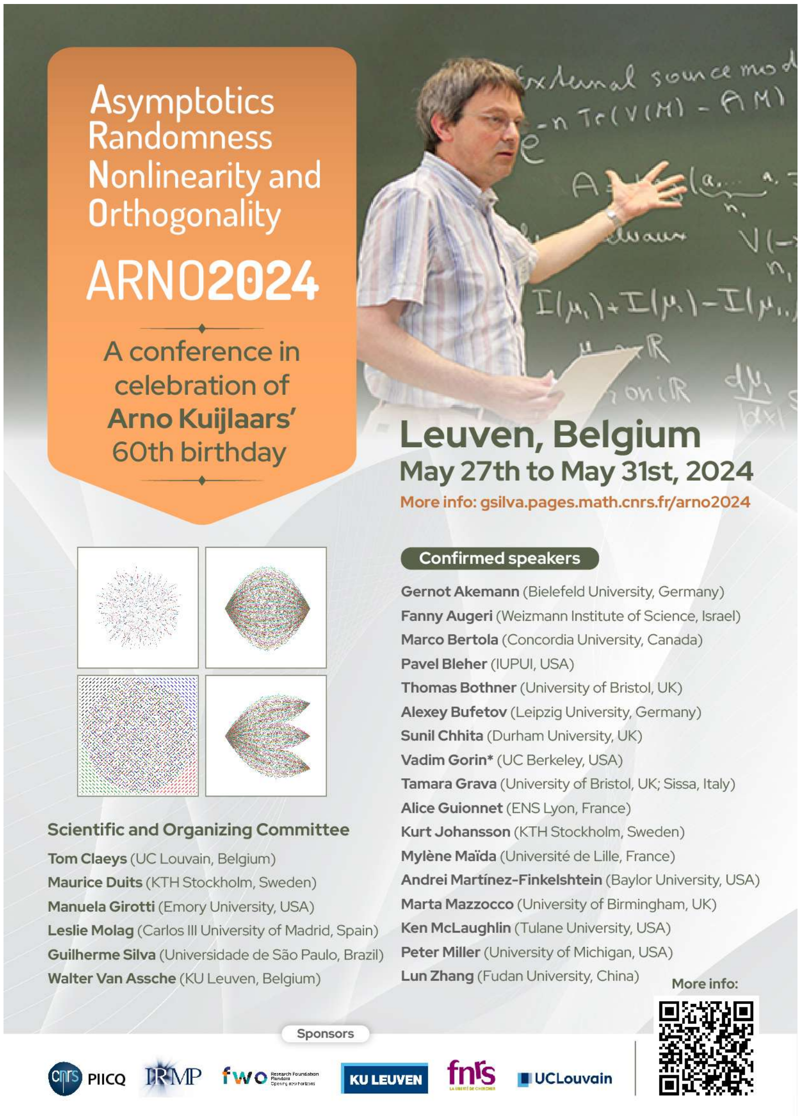


Figure 1: ARNO 2024 Conference Poster

From: Doron Lubinsky (lubinsky@math.gatech.edu)
Subject: Announcement: Constructive Functions 2025 in Nashville, Tennessee, USA

**Constructive Functions 2025**
Celebrating **Ed Saff's 80th birthday**
in conjunction with the **37th Shanks Lecture** by **Doron Lubinsky**

**Dates**: May 19-22, 2025
**Location**: Vanderbilt University, Nashville, Tennessee, USA

Figure 2: Ed Saff.

The conference will focus on several aspects of constructive function theory, including orthogonal polynomials, potential theory, discrete and continuous energy problems, special functions, polynomial inequalities, as well as various problems relating to optimization and efficiency. The aim is to stimulate collaboration and the exchange of ideas. **Early-career researchers (including students) as well as women and members of underrepresented groups are especially encouraged to attend.**

The conference will be held in conjunction with the **37th Annual Shanks Lecture**, to be given by Professor **Doron Lubinsky** (Georgia Institute of Technology). **The Lecture will be on Tuesday, May 20, 2025.** The prestigious Shanks Lecture Series is organized annually by the Department of Mathematics of Vanderbilt University, honoring Baylis and Olivia Shanks. The late Professor Baylis Shanks was chairman of the Department from 1955 through 1969. Previous Shanks Lecturers can be seen here.

The conference will also provide an opportunity to celebrate Professor **Ed Saff**'s recent 80th birthday. The conference will honor his many outstanding contributions to mathematics, his unselfish devotion to the promotion of approximation in the mathematical community, and his unceasing commitment to the education of the younger generation of mathematicians.

Organizing Committee:

- Doug Hardin (Vanderbilt University)
- Juliette LeBlond (INRIA Sophia Antipolis Méditerranée)
- Doron Lubinsky (Georgia Institute of Technology)
- Ryan Matzke (Vanderbilt University)
- Igor Pritsker (Oklahoma State University)
- Maya Stoyanova (Sofia University)
- Robert Womersley (University of New South Wales)
- Maxim Yattselev (IUPUI)

## Topic #3 ——— OP – SF Net 31.2 ——— March 15, 2024

From: Bernhard Beckermann (Bernhard.Beckermann@univ-lille.fr)
Subject: Announcement: Journées Approximation 2024 in Lille, France

First announcement/Call for participation
Journées Approximation 2024,
University of Lille, Lille, France
May 15–17, 2024

**Website**: https://indico.math.cnrs.fr/event/11523/

This is the sixth international meeting organized in Lille on approximation theory, numerical linear algebra and its applications.

Various topics will be covered, in particular orthogonal polynomials and rational approximation, numerical aspects of approximation, asymptotic analysis and Riemann-Hilbert problems, matrix functions, and others, with a special emphasis on approximation with exponential sums and their applications.

**Invited Speakers**:

- Laurent Baratchart (INRIA Nice, France)
- Jacob Christiansen (U Lund, Sweden)
- Dirk De Villiers (Stellenbosch U, South Africa)
- Ana Foulquié Moreno (U Alveiro, Portugal)
- Ferre Knaepkens (U Antwerp, Belgium)
- George Labahn (U Waterloo, Canada)
- Juliette Leblond (INRIA Nice, France)
- Wen-shin Lee (U Stirling, GB)
- Ana Filipa Loureiro (Kent U, GB)
- Bernard Mourrain (INRIA Nice, France)
- Miao-Jung Yvonne Ou (University of Delaware, US)
- Miguel Piñar (U Grenada, Espagne)
- Gerlind Plonka-Hoch (U Göttingen, Germany)
- Ramonika Sengupta (Eindhoven U, Netherlands)
- Nick Trefethen (Havard U, US)

- Walter Van Assche (KU Leuven, Belgium)
- Andre Weideman (Stellenbosch U, South Africa)
- Franck Wielonsky (Aix-Marseille U, France)
- Maxim Yattselev (Indiana U Purdue, US)

Registration is free but mandatory. Inscriptions are open now, until April 15, 2024.

There will be also a poster session, we kindly invite you to submit an abstract before April 15, 2024 to Ana.Matos@univ-lille.fr.

**Organization Committee**:

- Bernd Beckermann (Université de Lille)
- Ana C. Matos (Université de Lille)
- Laurent Smoch (Université du Littoral)

## Topic #4 ——— OP - SF Net 31.2 ——— March 15, 2024

From: Lothar Reichel (reichel@math.kent.edu)
Subject: Table of Contents: Special Volume dedicated to the 70$^{th}$ Birthday of **Lothar Reichel**

Table of Contents: Numerical Methods for Large Scale Problems,
A Special Volume dedicated to the 70$^{th}$ Birthday of **Lothar Reichel**

Electronic Transactions on Numerical Analysis (ETNA), vol. **59**, 2023:

This volume has been edited by the following faculty members of:
The University of Cagliari, Cagliari, Italy:
**Alessandro Buccini**, **Caterina Fenu**, **Luisa Fermo**, and **Giuseppe Rodriguez**.

Note: ETNA accepts software publications as well as historical papers.

link: https://etna.math.kent.edu/volumes/2021-2030/vol59/

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18. B. Sredojevic, Z. Milovanovic Jeknic, and D. Bojovic, pp. 342–355.
    A finite difference scheme for the approximation of the third initial boundary value parabolic problem

## Topic #5 ——— OP - SF Net 31.2 ——— March 15, 2024

From: Christian Berg (berg@math.ku.dk)
Subject: Memorial Contribution for **Bent Fuglede** (1925-2023) by **Christian Berg**

### Obituary for **Bent Fuglede**

Christian Berg, January 12, 2023.

Bent was PhD advisor to 6 students (including myself) at Københavns Universitet, København, Denmark.

Bent received the high school diploma from Skt. Jørgens Gymnasium in 1943, and he graduated from University of Copenhagen as mag. scient. and cand. mag. in 1948. After employment as scientific assistant at the Polytechnical Highschool, he was appointed associate professor at the Department of Mathematics at the University of Copenhagen. Having defended the dissertation “Extremal Length and Closed Extensions of Partial Differential Operators”, he was appointed professor at Denmark’s Technical Highschool in 1960.

Figure 3: Bent Fuglede (August 10, 1925 – July 12, 2023) is survived by his son Einar and daughter-in-law Dorthea and two grandsons. Bent’s wife of more than 60 years, Ólafia Einarsdóttir, died in 2017.

He returned to the University of Copenhagen as professor in 1965. He started by preparing excellent lecture notes for Mathematics 6, the major course in functional analysis that completed the first part of the cand. scient. study in mathematics. In the years that followed, he taught at all levels, always with a clear and well-worked presentation and with an easy-to-read blackboard script. His accompanying lecture notes often contained new results and elegant proofs. In 1992 he allowed himself to retire, but continued as an active researcher and professor emeritus at the Department of Mathematics and was active until shortly before his death.

Bent spent the years 1949–51 in the USA and stayed partly at Stanford University and partly at the Institute for Advanced Study in Princeton. It was during the visit at Stanford in November 1949 that “Fuglede’s Theorem”, was communicated by John von Neumann to the Proceedings of the National Academy of Sciences. This result was the first and probably most famous of Fuglede’s many theorems. It states, that if a bounded operator on a Hilbert space commutes with a normal operator, then it also commutes with the adjoint operator. It is a surprisingly deep result, which today is included in all advanced textbooks on operators in Hilbert spaces.

During Bent’s stay at the Institute for Advanced Study, he started collaboration with the American mathematician Richard V. Kadison on, among other things, determinant theory in infinite dimension. This led to Kadison coming to Copenhagen, and he gained a lifelong connection to Denmark, because he met his wife here. The strong position of operator algebra at our Institute is undoubtedly related to the influence of Kadison. The Fuglede-Kadison determinant continues to be used, e.g., by Lück in his development of $L^2$-cohomology and in the construction of the Brown measure for non-normal operators.

Subsequently, Bent studied partial differential operators, which led to the aforementioned doctoral dissertation, and also to research on potential theory. This theory deals with the solutions to Laplace’s equation, i.e. harmonic functions, Newton potentials, etc. In this subject, he quickly became a leading figure worldwide, and he often gave lectures at the potential theory seminar in Paris “Séminaire Brelot-Choquet-Deny”, named after the three leading French mathematicians in the subject. In the 1960s, Brelot and his students had developed an abstract potential theory called harmonic spaces. A harmonic space is a locally compact topological space, where a vector space of continuous real-valued functions is specified for each open subset, modelling the harmonic functions in open sets of a Euclidean space. Brelot had established a few fundamental axioms that these abstract spaces had to satisfy in order that many of the key results of potential theory could be deduced. The significance of the abstract theory was that one could obtain these key results for solution spaces to PDEs related to the Laplace operator by simply checking the axioms.

In harmonic spaces superharmonic functions can be introduced, but as in the classical theory, these are not always continuous, but only lower semi-continuous. In 1940, Henri Cartan got the idea to replace the classical Euclidean topology by a topology having more open sets, so that superharmonic functions became continuous. Cartan called this new topology the fine topology. The fine topology also appears in the theory of Brownian motion and similar stochastic processes, because a finely open set can be characterized as follows: If the process is in the finely open set at a specific time t0, then it remains there almost surely for a positive period of time. When Bent succeeded proving that the fine topology is connected and locally connected, he was inspired to construct what he called “finely harmonic functions in finely open sets”, and this was the start of a new era in potential theory and complex analysis. Bent published a comprehensive presentation of the theory in a Springer Lecture Notes: “Finely Harmonic Functions”, 1972, but already at the International Congress of Mathematicians (ICM) in Nice, 1970, he gave an invited lecture on it.

The topic became “hot”, and was further developed in the following years by Bent himself and with contributions from many others. Bent experienced the great honour that Heinz Bauer’s plenary lecture on potential theory at the ICM in Vancouver, 1974, was largely about Fuglede’s fine harmonic theory.

Just as harmonic functions in the plane are closely related to holomorphic functions, Bent succeeded in developing a rich theory for finely holomorphic functions in finely open sets of the complex plane. It also turned out that Fuglede’s theory was a natural continuation of Borel’s theory for “monogenic functions”, which was published as a monograph as early as 1917.

Bent’s work inspired a number of young mathematicians from abroad to new results, e.g., Terry Lyons (England) and Bernt Øksendal (Norway). Bent corresponded with a large number of mathematicians from home and abroad, and he was always generous with advice and improvements to manuscript drafts. He had a rare ability to penetrate the heart of a mathematical problem, and through this, he was often able to provide a surprising solution to the problem.

In 1974, Bent wrote a paper in the Journal of Functional Analysis: “Commuting self-adjoint partial differential operators and a group theoretic problem”. It has today received 298 citations. The work, which was inspired by a question from Irving Segal, associates a geometric property of an $n$-dimensional domain $G$ with a property of the Hilbert space $L^2(G)$. The result is later known as Fuglede’s Conjecture. The Danish mathematician Steen Pedersen from Ohio has published a number of papers on the subject and later Fields medal recipient Terence Tao published two papers: “Fuglede’s Conjecture holds for convex planar domains”, (2001) and “Fuglede’s Conjecture is false in 5 and higher dimensions”, (2003).

Bent decided to retire in the spring of 1992. He felt his health a little failing after a year’s stay at the Institute for Advanced Study in Princeton, where he was accompanied by Ólafia. Fortunately, his health turned out to be fine and Bent achieved an emeritus period of approx. 30 years, during which he was very active. Of the 114 of his works mentioned in Math. Sci. Net, approx. 40 is written after he became emeritus. During this period, he made significant contributions to many different areas, e.g., isoperimetric inequalities, Riemannian manifolds and moment problems. It should also be mentioned that he published a research monograph together with James Eells: “Harmonic maps between Riemannian Polyhedra”, Cambridge Tracts in Mathematics vol. 142, 2001. Together with

Natalia Zorii from Kyiv, he has since 2016 written 8 papers dealing with energy problems with respect to Riesz kernels, an impressive achievement for a person over 90 years of age.

Bent was a member of the Royal Danish Academy of Sciences and Letters, the Finnish Academy of Sciences and the Bavarian Academy of Sciences. On his retirement, the department celebrated him with a symposium. I had the pleasure of telling about Bent's mathematical results, and there were lectures by guests from several countries. The Danish Mathematical Society celebrated Bent's 70th birthday and named him an honorary member. For a number of years, he was a member of the editorial board of the journal Expositiones Mathematicae.

With his great and broad knowledge in many areas of mathematics, he was an obvious member of assessment committees at home and abroad. He has thereby had a great influence on how the department of mathematics developed.

Fuglede was active in bringing foreign capacities to the Institute on a one-year basis, including Masanori Kishi (Nagoya) and Mikhail Sodin (Kharkiv), both specialists in potential theory. Their appointments have had a great impact on my own research, and Sodin has subsequently been a regular guest at the Institute.

All theories have their time and then the interest in them decreases. This also happened to the abstract theory of harmonic spaces. Fuglede's theory for finely harmonic functions was formulated in Brelot's abstract theory. It was therefore difficult for new generations to utilize and appreciate Fuglede's theory, and this made Sodin emphasize at regular intervals that Bent should write a new presentation of the theory about finely harmonic functions, but only in connection with the classical potential theory.

Sodin succeeded in convincing Bent about the project, and he started to prepare a manuscript, which should include the results of his work on classical fine potential theory done in the period between 1970 and 2010. Bent had for many years corresponded with the Moroccan mathematician Mohamed El Kadiri about potential theory and invited him to collaborate on the book project "Classical Fine Potential Theory". They agreed on the collaboration and also to enlarge the scope to include results on finely holomorphic functions in one or several complex variables as well as a chapter on fine pluripotential theory. They also decided to include their joint recent research on Martin boundary for finely open sets. At the end of October 2023, the book was sent to Springer Nature.

The mathematical community can look forward to the publication of this fine closing of Bent Fuglede's career.

Glory be to his memory.

## Topic #6 ——— OP - SF Net 31.2 ——— March 15, 2024

From: OP-SF Net Editors
Subject: Remembrances of **Pascal Maroni** (1933-2024)

### Remembrances of Pascal Maroni (January 17, 1933-January 16, 2024).

Below are remembrances of Pascal Maroni from some of his colleagues and students:

**Alphonse Magnus**; **Claude Brezinski**; **Paco Marcellán**; **Zélia da Rocha**; **Ângela Macedo** and **Teresa Augusta Mesquita**; **Ana Loureiro**; **Lotfi Khériji**; and **Khalfa Douak** .

⋆ ⋆ ⋆

# My two cents on Pascal Maroni

Alphonse Magnus (alphonse.magnus@uclouvain.be)

**An instance of the Maroni's touch.**

Let $P_n$ and $\mathbb{P}_n$ be monic orthogonal polynomials related to the measures $d\mu(t)$ and

$$d\tilde{\mu}(t) = \frac{d\mu(t)}{t-c} + \kappa\delta(t-c)dt. \tag{1}$$

As

$$\int \mathbb{P}_n(t)P_m(t)d\mu(t) = \int \mathbb{P}_n(t)(t-c)P_m(t)\frac{d\mu(t)}{t-c} = \int \mathbb{P}_n(t)(t-c)P_m(t)d\tilde{\mu}(t) = 0, \tag{2}$$

when $m < n-1$ (the multiplication by $t-c$ kills the mass point at $c$!) and we have the quasi orthogonality representation $\mathbb{P}_n(x) = P_n(x)+c_nP_{n-1}(x)$. What are the $c_n$s? The $P_n$s are kernel polynomials, as $d\mu(t) = (t-c)d\tilde{\mu}(t)$ (same killing), so, by Christoffel–Darboux (Pascal would have written "Darboux–Christoffel")

$$(x-c)P_n(x) = \mathbb{P}_{n+1}(x) - \frac{\mathbb{P}_{n+1}(c)}{\mathbb{P}_n(c)}\mathbb{P}_n(x). \tag{3}$$

So,

$$(x-c)P_n(x) = P_{n+1}(x) + c_{n+1}P_n(x) - \frac{\mathbb{P}_{n+1}(c)}{\mathbb{P}_n(c)}(P_n(x)+c_nP_{n-1}(x)), \tag{4}$$

to compare to the $P_n$ recurrence relation, for instance

$$a_n = -\frac{\mathbb{P}_{n+1}(c)}{\mathbb{P}_n(c)}c_n, \tag{5}$$

or the equation for the $c_n$s

$$a_n(P_n(c)+c_nP_{n-1}(c)) = -(P_{n+1}(c)+c_{n+1}P_n(c))c_n, \tag{6}$$

etc.

Wimp & Kiesel [7, (1.9), (1.15)] ask when a combination of $P_n$ and $P_{n-1}$ satisfies a recurrence relation of the required form. We want

$$\mathbb{P}_{n+1}(x) = (x-\tilde{b}_n)\mathbb{P}_n(x) - \tilde{a}_n\mathbb{P}_{n-1}(x), \tag{7}$$

$$P_{n+1}(x) = (x-b_n)P_n(x) - a_nP_{n-1}(x), \tag{8}$$

therefore

$$\begin{aligned} 0 = \underbrace{P_{n+1}(x)}_{(x-b_n)P_n(x)-a_nP_{n-1}(x)} &+ c_{n+1}P_n(x) - (x-\tilde{b}_n)\{P_n(x)+c_nP_{n-1}(x)\} \\ &-\tilde{a}_n\left\{P_{n-1}(x)+c_{n-1}\left[P_{n-2}(x) = \frac{-P_n(x)+(x-b_{n-1})P_{n-1}(x)}{a_{n-1}}\right]\right\}. \end{aligned} \tag{9}$$

Whence [7, §2],

$$x-b_n+c_{n+1}-(x-\tilde{b}_n)+\frac{\tilde{a}_n c_{n-1}}{a_{n-1}}=-a_n-(x-\tilde{b}_n)c_n-\tilde{a}_n-\frac{\tilde{a}_n c_{n-1}(x-b_{n-1})}{a_{n-1}}\equiv 0. \tag{10}$$

They solve these equations, and more difficult ones too, by computer algebra.

Even the great Wolfgang Hahn has to struggle a short while with this problem [4, pp. 95–96], [5, (7), (8)], finding that $c_n$ is a ratio of solutions of the 3-term recurrence relation for $P_n$. Whereas Pascal solves the problem in a matter of seconds [6, p. 225], in showing "il faut et il suffit" that $\mathbb{P}_n$ be orthogonal to constants, so, that

$$c_n=-\frac{\int P_n(t)d\tilde{\mu}(t)}{\int P_{n-1}d\tilde{\mu}(t)}, \tag{11}$$

where $n=1,2,\dots$, and that

$$\int P_n(t)\frac{d\mu(t)}{t-c}, \tag{12}$$

is the numerator polynomial, or the first associated orthogonal polynomial $N_n(c)+$ a constant times $P_n(c)$.

**About forms.** Pascal writes $Lf$ for the more familiar (to whom?) measure writing

$$\int f(t)d\mu(t). \tag{13}$$

Let $Mf$ be our

$$\int \frac{f(t)}{t-c}d\mu(t), \tag{14}$$

with $(x-c)M$ applied to a test function $f$ gives $Lf$ of course, but Maroni's $(x-c)^{-1}Lf$ is NOT $Mf$, it gives $L$ applied to

$$\frac{f(t)-f(c)}{t-c}, \tag{15}$$

corresponding to

$$\frac{d\mu(t)}{t-c}-m_0\delta(t-c)dt, \tag{16}$$

where $m_0$ is the zeroth moment of $d\mu/(t-c)$. This is reminiscent of the formula of the numerator polynomials: in terms of Kiesel & Wimps's favorite representations by Stieltjes functions (my favorite too)

$$F(x)=\int\frac{d\mu(t)}{x-t}, \tag{17}$$

the numerator, or associated, polynomial $N_n$ is the polynomial part of

$$F(x)P_n(x)=\int P_n(x)d\mu(t)/(x-t)=\underbrace{\int\frac{P_n(t)-P_n(x)}{t-x}d\mu(t)}_{N_n(x)=P_{n-1}^{(1)}(x)}+\underbrace{\int\frac{P_n(t)}{x-t}d\mu(t)}_{Q_n(x)=\mathcal{O}(x^{-n-1})}, \tag{18}$$

so that

$$N_n(c)=(x-c)^{-1}LP_n. \tag{19}$$

Where are the forms coming from? They became often used since the 1980s [1], [2] and of course [6]. Shall we speak of a French touch? Pascal thought highly of Geronimus [3].

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## A luminous human being

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Pascal Maroni died in Paris on 16 January 2024, one day before his 91th anniversary. Pascal was born on 17 January 1933 in La Chaux-de-Fonds, Switzerland. When he was nineteen, he came to Paris to study at the *Faculté des sciences* where he obtained a *Licence de mathématiques* in 1957, followed by a diploma in celestial mechanics in 1958. Then he entered the *Centre national de la recherche scientifique* (CNRS) as a researcher in the *Institut Blaise Pascal*. There, in 1967, he defended a *Thèse de doctorat ès sciences mathématiques* under the guidance of René de Possel, one of the founders and first member of Bourbaki's group. The title was *Sur l'équation de Chandrasekhar*. When the institute was dissolved, Pascal joined the *Laboratoire d'analyse numérique* (now *Laboratoire Jacques-Louis Lions*) at the *Université Pierre et Marie Curie* in Paris where he remainder for his entire career becoming a *Directeur de recherche* at the CNRS in 1998.

Pascal first worked on numerical methods for integral equations with a special interest to Gaussian quadratures, which led him to the theory of orthogonal polynomials and special functions. He was close to Gérard Petiau, a renowned specialist of these functions. He began to organize a research group, which regularly met in his office, and to supervise theses in France, Spain, Tunisia and Portugal, where he was very much appreciated. He also managed to invite several foreign students for working stays with him in Paris. I've seen that they often remember these stays and are always grateful to Pascal for his invitations.

Over the years, Pascal built up an algebraic theory of orthogonal polynomials based on notions coming out from functional analysis, a work highly recognized by the international community, and often used without referring to him. He was one of the organizers of the first international congress on *Orthogonal Polynomials and Applications* held in Bar-le-Duc (France), 15–18 October 1984. He participated in a number of the annual French *Colloques d'analyse numérique*, and it is at the one held in Besse-en Chandesse (the village where the Bourbaki's group met in the 1950s) that I first met him in 1970. He also attended several international congresses where he gave invited talks. He published around one hundred papers.

Pascal also taught post-graduate courses at the *École centrale* in Paris, at the *Université Claude Bernard* in Lyon and in his own laboratory in Paris.

Pascal was a highly cultivated mathematician, but his knowledge extended far beyond this domain. He was interested in everything and was able to have discussions on almost all topics. He was a warm and charming person and a luminous human being.

We miss him.

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## In memoriam. Pascal Maroni (1933–2024)

Paco Marcellán (pacomarc@ing.uc3m.es)

Last January 16th we received from Claude Brezinski the sad news announcing that Pascal Maroni passed away in Paris just the day before his 91th birthday. We were first shocked with the death of André Ronveaux in December 2023 since they are both very good friends and scientific collaborators. They were involved in the organization of many meetings like the Bar-le-Duc, France, conference *Polynômes Orthogonaux et Applications*, held in 1984, which was the starting point of the OPSFA symposia series.

The first time I met Pascal was in Bar-le-Duc and between 1985 and 1996 we shared nice scientific and human times. I visited Université Pierre et Marie Curie in the period October 1987–February 1988 and I learned a lot in the weekly seminar conducted by Pascal, where his PhD students explained the progress of their Doctoral Dissertations, as well as in conversations during lunch time and after the activities in the university. We wrote two papers together and as well, as he visited Spain several times on the occasion of Spanish meetings on Orthogonal Polynomials, which contributed to consolidate a solid community on this topic in my country. During the years 1989 and 1990 our activity was supported by a Spain-France Bilateral Grant that allowed mobility of people of our teams as well as to organize joint meetings. I enjoyed the friendly atmosphere created by Pascal and without any doubt he contributed so much to my scientific career.

Let me now describe some relevant points of Pascal's scientific life. His Doctoral Dissertation with title, *La Résolution du système de Chandrasekhar*, was defended in 1967 in the Université de Paris-Sorbonne and supervised by René de Possel, head of the Laboratoire de Calcul Numérique in the Institut Blaise Pascal in Paris. In 1959 de Possel was appointed as the first Chair of Numerical Analysis in the Faculté des Sciences de Paris. It is also very relevant to point out that de Possel was a former PhD student of Constantin Carathéodory. Pascal was a member of the Centre National de la Recherche Scientifique (CNRS) in the Institut Blaise Pascal from 1959 and CNRS Directeur de Recherches from 1998 as well as a member of the Laboratoire Jacques-Louis Lions, the former Laboratoire d'Analyse Numérique in Université Pierre et Marie Curie, Paris VI.

According to MathSciNet Pascal authored 96 papers with 1452 citations. His most cited paper, with 217 citations, is P. Maroni, *Une théorie algébrique des polynômes orthogonaux. Application aux polynômes orthogonaux semi-classiques*, IMACS Ann. Comput. Appl. Math., 9 J. C. Baltzer A.G., Basel, 1991, 95–130. It was published in the Proceedings of the meeting held in Erice, Italy, in 1990, where Pascal was one of the Plenary speakers. Therein exists a wide approach to the properties of what are known today as semiclassical orthogonal polynomials based on linear functionals rather than integral representations, emphasizing the intrinsic relations that may exist between the functionals under consideration, either in the vector space of polynomial functions or in the vector

Figure 4: left-to-right: Paco Marcellán, Said Belmehdi, André Ronveaux, Charles-Michel Marle, Pascal Maroni, Lance L. Littlejohn, and Claude Brezinski.

space of formal power series isomorphic to it. This is a masterpiece written in French that can be considered as the main reference in the topic. Semiclassical orthogonal polynomials are characterized by the fact that the corresponding linear functional satisfies a Pearson equation and in this way, the concept of class is introduced. It allows to establish a hierarchy. In the bottom you have the classical orthogonal polynomials (Hermite, Laguerre, Jacobi and Bessel) as semiclassical of class 0. It constitutes an alternative to the Askey Tableau based on the hypergeometric character. As a generalization of semiclassical linear functionals, Pascal considered the so called Laguerre-Hahn classes and devoted many papers to such a topic.

Another important contribution is P. Maroni, *L'orthogonalité et les récurrences de polynômes d'ordre supérieur à deux*, Ann. Fac. Sci. Toulouse Math. (5) **10** (1989), no. 1, 105–139, a pioneering work on the so called $d$-orthogonal polynomials, i.e., where the orthogonality is defined in terms of vector linear functionals. This research line is a part of the theory of multiple orthogonal polynomials that has received the attention of many people interested on their structural properties and their applications in Padé-Hermite approximation and Gaussian quadrature rules.

On the other side, I would like to emphasize the powerful and successful activity by Pascal. In France he supervised 2 Thèses d'État, 6 Thèses de Doctorat and 13 Thèses de Troisième Cycle as well as being a co-supervisor of 4 Thèses d'État. In Tunisia he supervised 2 Thèse d'État and 6 Thèses d'Université and in Portugal, he co-supervised 4 Doctoral Theses with Zelia da Rocha, Universidade de Porto. We co-supervised in 1989 the Doctoral Dissertation *On differential properties of orthogonal polynomials on the unit circle* by Carmen Tasis, defended in Universidad de Cantabria, Santander, Spain. It was the result of a fruitful stay of Carmen in Université Pierre et Marie Curie in 1988 in the framework of a joint Project with Pascal dealing with the construction of semiclassical orthogonal polynomials on the unit circle. Notice that in the paper F. Marcellán, P. Maroni, *Orthogonal polynomials on the unit circle and their derivatives*, Constr. Approx. **7** (1991), no. 3, 341–348, we proved that the monomial polynomials associated with the Lebesgue measure on the unit circle, constitute the only "classical" orthogonal polynomials with respect to Hermitian linear functionals on the unit circle. Pascal was also a member of the Committees of several Doctoral Dissertations defended in Spanish universities from 1989 to 1996. On my side, I was a member of the Committee of the Thèse d'État *Formes linéaires et polynômes orthogonaux semiclassiques de classe* $s = 1$*. Description and classification* (249 pages), defended on October 1st , 1990, at Université Pierre et Marie Curie and supervised by Pascal. In the enclosed picture you can identify the other members of the Committee (A. Ronveaux, C. M. Marle, L. L. Littlejohn, and C. Brezinski).

From 2001, Pascal was involved in an ambitious collaboration research project with Portuguese mathematicians. He regularly visited Porto University as well as Tras-os-Montes e Alto Douro, Aveiro and Coimbra universities, contributing to consolidate a strong community working on orthogonal polynomials Portugal. A recognition of this commitment by Pascal was the Special Session on his life and work in the framework of the National meeting of the Sociedade Portuguesa de Matematica held in Bragança in July 2018, organized by Kenier Castillo (Universidade de Coimbra) and Ana Isabel Mendes (Instituto Politécnico de Leiria). I strongly recommend to read the contents of the contributions of this homage at the following link.

The memory of Pascal will remain in the mind and hearts of those who shared with him very nice moments.

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## Last seminar in Porto

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It was with deep sadness that I received the news of Pascal Maroni's passing away, through Claude Brezinski and Khalfa Douak, on the eve of what could have been his ninety-first birthday.

Pascal was in Porto last July on the occasion of the inauguration of the bibliographic exhibition that the Library of the Faculty of Sciences of the University of Porto dedicated to him [1], and also to participate in an event in which he gave a memorable talk entitled "Quelques remarques au sujet de la décomposition quadratique d'une suite de Laguerre" [2]. During that stay, I had the opportunity to discuss with him a recent work co-authored by our former Ph.D. students Ângela Macedo and Teresa Mesquita about general quadratic and cubic polynomial decompositions (defended in 2004 and 2010, respectively). The clarity of his mathematical reasoning and arguments was admirable despite his advanced age. In addition to these students, we had two others: Isabel Nicolau who worked on the inverse problem of the product of a regular form by a polynomial, and Ana Filipa Loureiro who devoted herself to the generalization of Hahn's problem and corresponding Appell sequences (defended in 2004 and 2009, respectively). I remember with appreciation the moments of discovery and scientific enthusiasm that we shared in supervising those theses. Also, I would like to mention so interesting seminars and communications that Pascal presented, not only at the University of Porto, but also at the Universities of Tràs-os-Montes e Alto Douro, Coimbra, and Aveiro.

I greatly value the joint work we developed, which dealt with the application of symbolic computations in obtaining connection coefficients for several kinds of orthogonal polynomials, and in the characterization of perturbed second-degree forms, in addition to other subjects we treated in partnership with our students. Finally, I refer to the course he taught in the first years of our collaboration, in which he explained the foundations of the algebraic theory of orthogonal polynomials he introduced in the mid-eighties [3]. Postgraduate students and colleagues from several departments regularly attended this excellent course.

Recognizing his outstanding scientific activity and contribution, the Portuguese Mathematics Society honored him at the 2018 meeting [4], and an interview was published in Gazeta da Matemática [5].

Figure 5: Pascal Maroni attending the “First Seminars of the Portuguese Group on Special Functions, Orthogonal Polynomials and Applications” in the Library of the Faculty of Sciences of the University of Porto, Portugal, on July, 12th 2023.

In the same year, he was also honored by the University of Porto, which recorded a live interview, so now we can all remember and appreciate his image and reports on his scientific journey [6].

I had the privilege of collaborating with Pascal Maroni for more than 20 years [7] and benefiting from his deep mathematical knowledge and experience in the academic world. For all I have learned from him, I will be eternally grateful.

Figure 6: From left to right: Pascal Maroni, Ana Filipa Loureiro, and Zélia da Rocha some moments after Filipa's doctoral thesis defense at the Departement of Mathematics of the Faculty of Sciences of the University of Porto, in 2009.

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## The relevance of second-degree forms according to Pascal Maroni

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Second-degree forms are regular forms whose formal Stieltjes function is

$$S(u)(z) := -\sum_{n\geq 0} \frac{(u)_n}{z^{n+1}} ,$$

where $(u)_n = \langle u, x^n \rangle$ denote the moments of the form $u$, satisfies the following quadratic equation

$$B(z)S^2(u)(z) + C(z)S(u)(z) + D(z) = 0 ,$$

with certain polynomials $B$, $C$, and $D$ [4].

These forms possess the remarkable property of being semi-classical because they satisfy an affine Stieltjes equation [4, 5]

$$\widehat{A}(z)S'(u)(z) = \widehat{C}(z)S(u)(z) + \widehat{D}(z) , \tag{20}$$

whose coefficients can be determined from $B$, $C$, and $D$ as follows

$$\begin{aligned}
\widehat{A}(z) &= B(z)\{C^2(z) - 4B(z)D(z)\} ,\\
\widehat{C}(z) &= 2B(z)\{B'(z)D(z) - D'(z)B(z)\} + C(z)\{C'(z)B(z) - B'(z)C(z)\} ,\\
\widehat{D}(z) &= B(z)\{C'(z)D(z) - D'(z)C(z)\} + D(z)\{C'(z)B(z) - B'(z)C(z)\} .
\end{aligned}$$

We recall that equation (20) constitutes one of the characterizations of semi-classical forms [4]. Consequently, the polynomial sequence $\{P_n(x)\}_{n\geq 0}$ orthogonal with respect to $u$ verifies a linear differential equation of order two of the following type

$$J(x;n)P''_{n+1}(x) + K(x;n)P'_{n+1}(x) + L(x;n)P_{n+1}(x) = 0 , \ n \geq 0 ,$$

with $\deg J(\cdot;n) \le 2s+2$, $\deg K(\cdot;n) \le 2s+1$, $\deg L(\cdot;n) \le 2s$, $n \ge 0$, being $s$ the class of the form. These coefficients are given by

$$
\begin{aligned}
&J(x;n) = \widehat{A}(x)\widehat{D}_{n+1}(x) \ , \\
&K(x;n) = \widehat{C}_0(x)\widehat{D}_{n+1}(x) - W(\widehat{A},\ \widehat{D}_{n+1})(x) \ , \\
&L(x;n) = W\left(\frac{1}{2}(\widehat{C}_{n+1} - \widehat{C}_0), \widehat{D}_{n+1}\right)(x) - \widehat{D}_{n+1}(x)\sum_{\nu=0}^{n}\widehat{D}_\nu(x) \ ,
\end{aligned}
$$

where $W(f,g)$ represents the Wronskian of $f$ and $g$ [4].

Furthermore, second-degree forms constitute a closed set for the most common transformations of sequences such as shifting, association, and perturbation, as well as, for the inverse of a form, and the product of a form by a polynomial [4]. In other words, a transformation of a second-degree form by one of these operations remains of second-degree and consequently, it is semi-classical.

Among the classical sequences (Hermite, Laguerre, Jacobi, and Bessel), only certain Jacobi forms, $J(k-\frac{1}{2}, l-\frac{1}{2})$, $k+l \ge 0$, $k,\ l \in \mathbb{Z}$, are of second-degree, in which the four families of Chebyshev are included [1]. Therefore, it is possible to construct new semi-classical sequences from those second-degree families by applying the transformations mentioned (see, for example, [6, 7]). In more complex cases, it may be necessary to use symbolic computations to obtain closed formulas for the coefficients of the differential equation [2, 3]. On the other side, identifying a given sequence as transformed of a second-degree family may facilitate the study of its properties.

At last, it is worth mentioning that second-degree forms lie at the boundary between semi-classical and Laguerre-Hahn forms as they belong to both sets [4].

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# Remembrance of Pascal Maroni: Quadratic and cubic decompositions of a given monic polynomial sequence

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The quadratic decomposition of a symmetric orthogonal polynomial sequence into two polynomial sequences was firstly proposed by Carlitz and Chihara [1, 3].

In the early 1990s, Pascal Maroni dedicated two papers to the study of a full quadratic decomposition (QD) of an orthogonal monic polynomial sequence (monic PS) [8, 10], following the work presented by Laura & Theodore Chihara [4]. Later on, in collaboration with Ângela Macedo, he addressed several questions regarding a generalization of that quadratic decomposition that is defined through a monic quadratic polynomial [14]

$$\omega(x) = x^2 + px + q\,,\ \ p, q \in \mathbb{C}\,,$$

and a constant $a \in \mathbb{C}$, so that we can always associate to any given monic PS $\{W_n\}_{n\geq 0}$ two further monic PS $\{P_n\}_{n\geq 0}$ and $\{R_n\}_{n\geq 0}$ and two additional sequences of polynomials $\{a_n\}_{n\geq 0}$ and $\{b_n\}_{n\geq 0}$ by means of the following unique QD:

$$\begin{aligned} W_{2n}(x) &= P_n(\omega(x)) + (x-a)a_{n-1}(\omega(x)),\\ W_{2n+1}(x) &= b_n(\omega(x)) + (x-a)R_n(\omega(x)), \end{aligned}$$

where $\deg\left(a_n(x)\right) \leq n\,,\ \deg\left(b_n(x)\right) \leq n\,,\ n \geq 0\,,$ and $a_{-1}(x) = 0\,.$

In his research work, the $d$-orthogonal polynomials were another subject of high interest (e.g., [7], [9]), namely, we may read about some families of $d$-orthogonal polynomials in the work with Khalfa Douak [5, 11, 12, 13]. Most in particular, in some of these references we can find, by means of the notion of $d$-symmetry, a natural cubic decomposition for the $2$-orthogonal and $2$-symmetric polynomial sequences.

Not long after, it was explored a general cubic decomposition [15] that provides nine polynomial sequences divided into two sets: three principal components and six secondary components, together with six parameters, generalizing [6] and revisiting [2]. More precisely, fixing a monic cubic polynomial

$$\varpi(x) = x^3 + px^2 + qx + r,$$

and three constants $a, b$ and $c$, it was proved that for any monic PS $\{W_n\}_{n\geq 0}$, there are three monic PS $\{P_n\}_{n\geq 0}$, $\{Q_n\}_{n\geq 0}$ and $\{R_n\}_{n\geq 0}$, and six other sequences of polynomials $\{a_n^1\}_{n\geq 0}$, $\{a_n^2\}_{n\geq 0}$, $\{b_n^1\}_{n\geq 0}$, $\{b_n^2\}_{n\geq 0}$, $\{c_n^1\}_{n\geq 0}$ and $\{c_n^2\}_{n\geq 0}$, such that

$$\begin{aligned} W_{3n}(x) &= P_n(\varpi(x)) + (x-a)a_{n-1}^1(\varpi(x)) + (x-b)(x-c)a_{n-1}^2(\varpi(x)),\\ W_{3n+1}(x) &= b_n^1(\varpi(x)) + (x-a)Q_n(\varpi(x)) + (x-b)(x-c)b_{n-1}^2(\varpi(x)),\\ W_{3n+2}(x) &= c_n^1(\varpi(x)) + (x-a)c_n^2(\varpi(x)) + (x-b)(x-c)R_n(\varpi(x)), \end{aligned}$$

with $a_{-1}^1(x) = a_{-1}^2(x) = b_{-1}^2(x) = 0$; $\deg\left(a_{n-1}^1(x)\right) \leq n-1$, $\deg\left(a_{n-1}^2(x)\right) \leq n-1$, $\deg\left(b_n^1(x)\right) \leq n$, $\deg\left(b_{n-1}^2(x)\right) \leq n-1$, $\deg\left(c_n^1(x)\right) \leq n$ and $\deg\left(c_n^2(x)\right) \leq n$, for $n \geq 0$.

As established by the research contributions of many authors over the last decades, the quadratic and the cubic decompositions, either with a simple or parameterized layout, became fruitful methods of constructing new polynomial sequences featuring interesting properties depending on the specifics of the given sequence (e.g., [16]).

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## The magic of the algebraic theory of orthogonal polynomials

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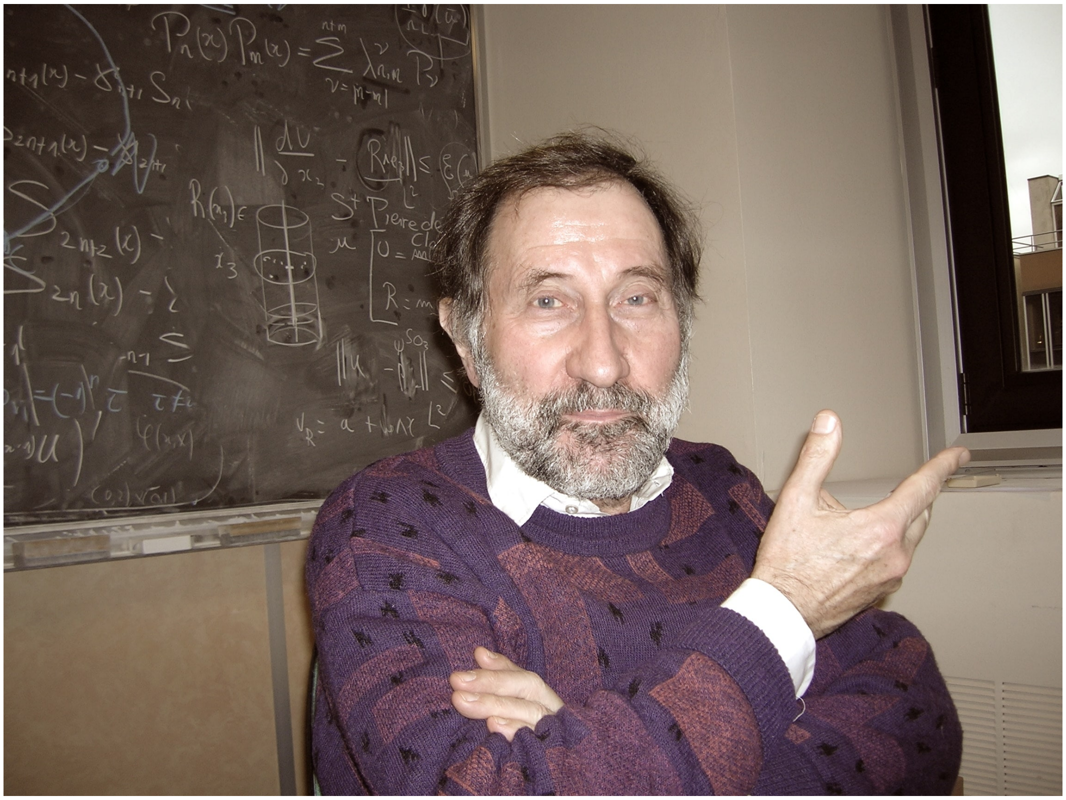

Figure 7: Pascal Maroni in his office at LJLL in Paris VI.

Pascal was my PhD supervisor and a great friend. Pascal was an inspirational source of knowledge scientifically and as person. A remarkably cultivated man, an authentic character, with a judicious mind, who payed an enormous attention to detail and to the context. After meeting Pascal during my masters at the University of Porto, I then embarked on a joyful journey of research in orthogonal polynomials and special functions. Pascal was an excellent supervisor, giving the freedom of exploring ideas whilst guiding by asking questions and listening to ideas, patiently discussing whether these would lead to interesting problems. I cherish all our discussions dearly and gratefully.

Starting in the 80s and then throughout the 90s, Pascal developed the algebraic theory of orthogonal polynomials. Several of the key ideas were depicted in the french written paper [6]. The algebraic approach was thoroughly applied in the study of extensions and generalisations of standard orthogonality, most relevantly, weak orthogonality, quasi-orthogonality and $d$-orthogonality, whilst embedding the concepts of Hahn-classical, semiclassical, Laguerre-Hahn polynomials (and forms), among others. The techniques rely heavily on functional operations, and on a joyful interplay between the polynomial sequence and the corresponding dual sequence. Pascal's prolegomenon to semiclassical polynomials [4] (which was completed soon after [3]) is a landmark for several emerging approaches that followed after. In [4] Pascal describes semiclassical polynomials as orthogonal polynomials such that the sequence of its derivatives is quasi-orthogonal. From here he goes on to obtain a structural relation and then a second order differential equation, whose coefficients are recursively defined. One feature of Pascal's work is the mathematical detail in his contributions, setting all the notations, definitions and results extremely precise. One may eventually see his papers as potentially challenging to read, but undoubtedly resourceful. A good, but very short, summary of the outcome of playing in the Maroni's linear functional wonderland is [7].

Research on higher order recurrence relations was another theme that Pascal contributed considerably, with a debut in [1] followed by [5]. The contributions then followed in some other papers jointly authored with Khalfa Douak, where they gave a characterisation of Hahn-classical $d$-orthogonal polynomials. These are essentially multiple orthogonal polynomials of type II for $d$ measures and whose indexes lie on the step line. The corresponding type I multiple orthogonal functions can be mapped to the dual sequence of $d$-orthogonal polynomials. Along the way they offered an array of techniques and tools to address several characterisation and classification problems in the world of $d$-orthogonal polynomials.

Pascal was always taking his analysis very seriously, and is calculations in his neat handwriting were incredibly accurate (without using any symbolic computational aids). He would normally check and recheck his work to the fine detail.

On a personal level, Pascal was a very cultivated gentleman, with a great sense of humour and always ready for a *bonne soirée*. A very kind and generous soul and a fantastic friend. I feel fortunate to have met Pascal and I cherish all the moments we spent together.

Figure 8: From left to right: Isabel Nicolau, Ana F. Loureiro, and Pascal Maroni in a congress at the University of Tràs-os-Montes e Alto Douro, Vila Real, Portugal.

**Thank you Pascal for all your contributions and all that you offered us.**

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Figure 9: Ana Loureiro and Pascal Maroni in Paris, June 2023.


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## In loving memory of my Master Pascal Maroni (1933–2024)


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Tunis El Manar University


In loving memory of my Master Pascal Maroni (1933–2024),
Founder of the research group in orthogonal polynomials at Gabès University, Tunisia in 1995.

In the realm of orthogonal polynomials, Pascal Maroni stands as a revered figure, leaving behind a legacy marked by his profound contributions and unwavering passion for mathematical exploration.

As a dedicated researcher, Maroni's work illuminated the intricate world of orthogonal polynomials, unraveling their mathematical nuances and expanding the boundaries of understanding. Beyond the equations and theorems, his Tunisian PhD students fondly remember his warm spirit and genuine enthusiasm for sharing knowledge. Also, they often reflect on the collaborative moments spent with Maroni, where his insights and kindness created an environment of intellectual camaraderie. Pascal Maroni's loving memory endures not only through the enduring impact of his research but also in the hearts of those who were fortunate to witness his brilliance and experience his generous mentorship in the fascinating domain of orthogonal polynomials.

On behalf of his Tunisian PhD students.
Peace to your soul, dear master.

⋆ ⋆ ⋆

## To remember Pascal Maroni

Khalfa Douak (khalfa_douak@yahoo.fr)

I first met Pascal Maroni over forty years ago, when he was Director of Research at CNRS, and member of the Laboratoire d'Analyse Numérique at the Université Pierre et Marie Curie-Paris VI (now Laboratoire J.L. Lions Sorbonne Université Campus Pierre et Marie Curie Paris). As one of his PhD students, I got to know him better over the years, and even more so as I became a close collaborator. Together we maintained a research collaboration until his death. Our close relationship over the years turned into a great friendship.

Figure 10: Pascal Maroni and Khalfa Douak in Pascal's office at LJLL, 2005.

Pascal Maroni was first of all my supervisor for my P.h.D. thesis, defended in 1988, then my mentor who constantly supported and encouraged me to pursue research in the field of orthogonal polynomials and special functions, and more particularly in the recent field of d-orthogonality, which still remains to be explored. Benefiting from his advice and the many fruitful discussions I had with him, I continued my research work until I submitted my Habilitation à Diriger des Recherches in 2003.

Throughout his scientific career, P. Maroni has supervised numerous theses in France, Tunisia and Portugal. In France, he organized a weekly seminar in his office to lead a working group on orthogonal polynomials and special functions. I gladly took part in most of these seminars, and did so for many years. On the other hand, and within the same framework, he had often welcomed some of the specialists in the field for joint work sessions that were varied and rich in exchanges. On many occasions, these meetings led to close collaboration on various scientific projects, resulting in numerous publications. With his paper L'orthogonalité et les récurrences de polynômes d'ordre supérieur à deux, Ann. Fac. Sci. Toulouse Math. (5) 10 (1989), no. 1, 105–139, P. Maroni is considered as a precursor of the d-orthogonality notion. All our joint work is carried out within this framework.

Personally, after having introduced me to research, Pascal Maroni has been a constant source of encouragement and advice to me. I have learned a great deal from him. So, I'd like to take this opportunity to express my deep gratitude to him once again, and above all to pay him a great tribute. I always remember him as a deeply human man, warm and attentive to others, open to discussions on the various questions we asked ourselves or the problems we had to deal with together.

## Topic #7 ——— OP – SF Net 31.2 ——— March 15, 2024

From: OP-SF Net Editors
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during January and February 2024. This list has been separated into two categories.

### OP-SF Net Subscriber E-Prints

http://arxiv.org/abs/2401.00556
The evaluation of a definite integral by the method of brackets illustrating its flexibility
Ivan Gonzalez, John Lopez Santander, Victor H. Moll

http://arxiv.org/abs/2401.00752
A note on Laguerre truncated polynomials and quadrature formula
Juan C. García-Ardila, Francisco Marcellán

http://arxiv.org/abs/2401.01971
Bordered and Framed Toeplitz and Hankel Determinants with Applications to Integrable Probability
Roozbeh Gharakhloo, Karl Liechty

http://arxiv.org/abs/2401.02515
Limits of Bessel functions for root systems as the rank tends to infinity
Dominik Brennecken, Margit Rösler

http://arxiv.org/abs/2401.02820
Quasi-Jacobi forms, Appell-Lerch functions, and false theta functions as $q$-brackets of functions on partitions
Kathrin Bringmann, Jan-Willem van Ittersum, Jonas Kaszian

http://arxiv.org/abs/2401.03608
The ultraspherical rectangular collocation method and its convergence

Thomas Trogdon

http://arxiv.org/abs/2401.04586
Quasi-exactly solvable potentials in Wigner-Dunkl quantum mechanics
C. Quesne

http://arxiv.org/abs/2401.08470
Hypergeometric Solutions of Linear Difference Systems
Moulay Barkatou, Mark van Hoeij, Johannes Middeke, Yi Zhou

http://arxiv.org/abs/2401.08481
Determinant evaluations inspired by Di Francesco's determinant for twenty-vertex configurations
Christoph Koutschan, Christian Krattenthaler, Michael Schlosser

http://arxiv.org/abs/2401.08514
Beyond Weisfeiler-Lehman: A Quantitative Framework for GNN Expressiveness
Bohang Zhang, Jingchu Gai, Yiheng Du, Qiwei Ye, Di He, Liwei Wang

http://arxiv.org/abs/2401.09562
On an identity by Ercolani, Lega, and Tippings
Maxim L. Yattselev

http://arxiv.org/abs/2401.10130
Biorthogonal measures, polymer partition functions, and random matrices
Mattia Cafasso, Tom Claeys

http://arxiv.org/abs/2401.12110
Differentiation of integral Mittag-Leffler and integral Wright functions with respect to parameters
Alexander Apelblat, Juan Luis González-Santander

http://arxiv.org/abs/2401.12083
Evaluations of $\sum_{k=1}^{\infty} \frac{x^k}{k^2\binom{3k}{k}}$ and related series
Zhi-Wei Sun, Yajun Zhou

http://arxiv.org/abs/2401.12430
Enumerating Seating Arrangements that Obey Social Distancing
George Spahn, Doron Zeilberger

http://arxiv.org/abs/2401.14099
On a transformation of triple $q$-series and Rogers-Hecke type series
Zhi-Guo Liu

http://arxiv.org/abs/2401.14197
Proof of conjectures on series with summands involving $\binom{2k}{k}8^k/(\binom{3k}{k}\binom{6k}{3k})$
Zhi-Wei Sun, Yajun Zhou

http://arxiv.org/abs/2401.16314
Creative Telescoping for Hypergeometric Double Sums
Peter Paule, Carsten Schneider

http://arxiv.org/abs/2401.16671
Resurgence in the transition region: The incomplete gamma function
Gergő Nemes

http://arxiv.org/abs/2401.17209
Unveiling new perspectives of hypergeometric functions using umbral techniques
Giuseppe Dattoli, Mehnaz Haneef, Subuhi Khan, Silvia Licciardi

http://arxiv.org/abs/2401.17220
Leading coefficient in the Hankel determinants related to binomial and $q$-binomial transforms
Shane Chern, Lin Jiu, Shuhan Li, Liuquan Wang

http://arxiv.org/abs/2401.17446
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Robert E. Gaunt, Siqi Li

http://arxiv.org/abs/2401.17674
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Stefan Steinerberger

http://arxiv.org/abs/2402.03381
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Kathrin Bringmann, William Craig, Jan-Willem van Ittersum, Badri Vishal Pandey

http://arxiv.org/abs/2402.08378
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## Topic #8 ——— OP - SF Net 31.2 ——— March 15, 2024

From: OP-SF Net Editors
Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 31.3 should be sent by May 1, 2024.

OP-SF NET is the electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials (SIAG/OPSF). We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to OP-SF Talk which is currently managed and moderated by Howard Cohl (howard.cohl@nist.gov). Anyone wishing to be included in the mailing list (SIAG/OPSF members and non-members alike) should send an email expressing interest to him. Bonita Saunders also posts the Newsletter through SIAM Engage (SIAG/OPSF) which is received by all SIAG/OPSF members.

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Sarah Post, OP-SF NET co-editor
Bonita Saunders, Webmaster and SIAM Engage (SIAG/OPSF) moderator

*As of the date of the publication of OP-SF NET 31.2, the SIAG/OPSF elections have not occurred.

## Topic #9 ——— OP - SF Net 31.2 ——— March 15, 2024

From: OP-SF Net Editors
Subject: Thought of the Month by **Pascal Maroni**

“The basic idea in what follows is to work directly on linear forms and not on integral representations; in other words, to favor the intrinsic relationships that can exist between the forms considered, either by placing oneself in the dual of the vector space of polynomial functions or by placing oneself in the vector space of formal series which is isomorphic to it.”

"L'idée de base dans ce qui suit, consiste à travailler directement sur les formes linéaires et non pas sur des représentations intégrales; autrement dit, de privilégier les relations intrinsèques qui peuvent exister entre les formes envisagées, soit en se plaçant dans le dual de l'espace vectoriel des fonctions polynomiales, soit en se plaçant dans l'espace vectoriel des séries formelles qui lui est isomorphe."

**P. Maroni**, Une théorie algébrique des polynômes orthogonaux. Application aux polynômes orthogonaux semi-classiques (An algebraic theory of orthogonal polynomials. Applications to semi-classical orthogonal polynomials), in: C. Brezinski, et al. (Eds.), Orthogonal Polynomials and their Applications (Erice, 1990), in: IMACS (International Association for Mathematics and Computers in Simulation) Annals on Computing and Applied Mathematics, **9**, Baltzer, Basel, 1991, pp. 95–130 (in French).