Applied portfolio analysis

Lecture II

Fundamentals in optimal portfolio choice

How do we choose the optimal allocation?

- What inputs do we need?
- How do we choose them?
- How easy is to get exact solutions in arbitrary settings?
- What approximations can we use?
- What happens in practice?

Inputs

Investor input

- Current allocation
- Investment opportunities
- Objectives (often multiple)
- Indices of satisfaction
- Investment horizon

Market input

- Asset prices (joint distribution)/ asset returns
- Trading/implementation costs
- Benchmarks
- Constraints

Example

Investor

- Objectives
  absolute wealth: $X_T^{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_T \cdot \mathbf{P}_T$
  net profits: $\tilde{X}_T^{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_T \cdot (\mathbf{P}_T - \mathbf{P}_0)$

- Satisfaction indices
  $\mathcal{S}(\boldsymbol{\alpha}) = CE(\boldsymbol{\alpha}) = u^{-1}(E(u(X^{\boldsymbol{\alpha}})))$ (primary)
  $\tilde{\mathcal{S}}(\boldsymbol{\alpha}) = -Var_c(\boldsymbol{\alpha}) = Q_{\tilde{X}_T^{\boldsymbol{\alpha}}}(1-c)$ (secondary)

- Risk preferences: $u(x) = -e^{-\frac{1}{\zeta}x}$
  $\zeta \in \left[\zeta^l, \zeta^u\right]$ risk propensity/risk tolerance

Market

Asset prices: $\mathbf{P}_T \sim \mathcal{N}(\xi, \mathbf{\Phi})$

ξ : expected values and $\mathbf{\Phi}$: covariance matrix

$$\mathbf{M} \equiv \mathbf{P_T} \sim \mathcal{N}(\xi, \mathbf{\Phi}) \qquad \tilde{\mathbf{M}} \equiv \mathbf{P_T} - \mathbf{P_0} \sim \mathcal{N}\xi - \mathbf{P_0}$$

$$X_T^{\boldsymbol{\alpha}} \sim \mathcal{N}\left(\xi' \boldsymbol{\alpha}, \boldsymbol{\alpha}' \mathbf{\Phi} \boldsymbol{\alpha}\right) \qquad \tilde{X}_T^{\boldsymbol{\alpha}} \sim \mathcal{N}\left((\xi - \mathbf{P}_0)' \boldsymbol{\alpha}, \boldsymbol{\alpha}' \mathbf{\Phi} \boldsymbol{\alpha}\right)$$

Indices of satisfaction

$$CE(\boldsymbol{\alpha}) = \xi' \boldsymbol{\alpha} - \frac{1}{2\zeta} \boldsymbol{\alpha}' \mathbf{\Phi} \boldsymbol{\alpha} \qquad \zeta > 0$$

$$Var_c(\boldsymbol{\alpha}) = (\mathbf{P}_0 - \xi)' \cdot \boldsymbol{\alpha} + \sqrt{2 \boldsymbol{\alpha}' \mathbf{\Phi} \boldsymbol{\alpha}}\, \mathrm{erf}^{-1}(2c - 1)$$

Constraints

- No free-lunches $\longleftrightarrow$ Budget constraint (in case of no transaction costs)
- VaR cannot exceed a given "bugdet at risk" (γ - fraction of initial endowment)

These requirements lead to the following state/control constraints

$$\mathcal{C}_1 : \mathbf{P}_0^{'} \cdot \boldsymbol{\alpha} = x_0$$

$$\mathcal{C}_2 : Var_c(\boldsymbol{\alpha}) \leq \gamma x_0$$

Feasible set of allocations

$$e \equiv \xi' \cdot \boldsymbol{\alpha} \qquad d \equiv \sqrt{\boldsymbol{\alpha}' \boldsymbol{\Phi} \boldsymbol{\alpha}}$$

- The budget constraint is satisfied to the right of the hyperbola

$$d^2 \geq \frac{A}{D} e^2 - \frac{2 x_0 B}{D} e + \frac{x_0^2 C}{D}$$

where the market parameters are

$$A \equiv \mathbf{P}_0' \boldsymbol{\Phi}^{-1} \mathbf{P}_0 \qquad B \equiv \mathbf{P}_0' \boldsymbol{\Phi}^{-1} \xi$$

$$C \equiv \xi' \boldsymbol{\Phi}^{-1} \xi \qquad D = AC - B^2$$

- The VaR constraint is satisfied by all points above the straight line

$$e \geq (1 - \gamma)\, x_0 + \sqrt{2}\, \mathrm{erf}^{-1} (2c - 1)\, d$$

Optimal allocation

$$\boldsymbol{\alpha}^* = \arg\max_{\substack{\mathbf{P}_0' \cdot \boldsymbol{\alpha} = x_0 \\ Var_c(\boldsymbol{\alpha}) \leq \gamma x_0}} \{CE(\boldsymbol{\alpha})\}$$

It turns out that the **optimal allocation** and the **objective at the optimum** are given by

$$\boldsymbol{\alpha}^* = \zeta \boldsymbol{\Phi}^{-1} \xi + \frac{x_0 - \zeta \mathbf{P}_0' \boldsymbol{\Phi}^{-1} \xi}{\mathbf{P}_0' \boldsymbol{\Phi}^{-1} \mathbf{P}_0} \boldsymbol{\Phi}^{-1} \mathbf{P}_0$$

$$CE(\boldsymbol{\alpha}^*) = \frac{\zeta}{2} \xi' \boldsymbol{\Phi}^{-1} \xi + \frac{1}{2} \left(\frac{x_0 - \zeta \mathbf{P}_0' \boldsymbol{\Phi}^{-1} \xi}{\mathbf{P}_0' \boldsymbol{\Phi}^{-1} \mathbf{P}_0} \right) \xi' \boldsymbol{\Phi}^{-1} \mathbf{P}_0 - \frac{1}{2\zeta} \frac{\left(x_0 - \zeta \mathbf{P}_0' \boldsymbol{\Phi}^{-1} \xi \right)^2}{\mathbf{P}_0' \boldsymbol{\Phi}^{-1} \mathbf{P}_0}$$

Recall: Investor input (CE, ζ, x_0) and Market input $(\mathcal{N}, \xi, \boldsymbol{\Phi})$

Limitations in specifying an optimum

Static cases

- Explicit solutions rarely exist
- Numerical solutions can be prohibitely difficult and 'expensive' if, for example, the objective fails to be concave and/or constraints are not of cone type
- Approximate solutions need to be constructed
- Stringent assumptions about the market returns might be needed

Mean-variance analysis

Mean - Variance analysis

Fundamental assumption: $\mathcal{S}(\boldsymbol{\alpha}) \simeq \mathcal{G}\left(E\left(X^{\boldsymbol{\alpha}}\right), Var\left(X^{\boldsymbol{\alpha}}\right)\right)$

Example: $CE(\boldsymbol{\alpha}) \simeq E\left(X^{\boldsymbol{\alpha}}\right) - \frac{A(E(X^{\boldsymbol{\alpha}}))}{2} Var\left(X^{\boldsymbol{\alpha}}\right)$

Optimization algorithm

- Determine the one-parameter family

$$\boldsymbol{\alpha}(v) = \arg\max E\left(X^{\boldsymbol{\alpha}}\right); \ \ \boldsymbol{\alpha} \in \mathcal{C},\ Var(X^{\boldsymbol{\alpha}}) = v$$

- Determine the optimum

$$\boldsymbol{\alpha}^* = \boldsymbol{\alpha}(v^*) = \arg\max_{v \geq 0} \mathcal{S}(\boldsymbol{\alpha}(v))$$

Mean - Variance optimization problem

$$E\left(X^{\boldsymbol{\alpha}}\right) = \boldsymbol{\alpha}^{'} E\left(\mathrm{M}\right)$$

$$Var\left(X^{\boldsymbol{\alpha}}\right) = \boldsymbol{\alpha}^{'} Cov\left(\mathrm{M}\right) \boldsymbol{\alpha}$$

$E\left(\mathrm{M}\right)$, $Cov\left(\mathrm{M}\right)$:Expected value and covariance of market vector

$$\boldsymbol{\alpha}\left(v\right) = \operatorname{arg\,max} E\left(X^{\boldsymbol{\alpha}}\right); \;\; \boldsymbol{\alpha} \in \mathcal{C}, \quad \boldsymbol{\alpha}^{'} Cov\left(\mathrm{M}\right) \boldsymbol{\alpha} = v$$

Example I

Efficient frontier with affine constraints

$$\boldsymbol{\alpha}(v) = \arg\max E\left(X^{\boldsymbol{\alpha}}\right); \quad \boldsymbol{\alpha}' d = c, \;\; Var\left(X^{\boldsymbol{\alpha}}\right) = v$$

d a vector not colinear with $E(\mathbf{M})$

$$\boldsymbol{\alpha}(e) = \boldsymbol{\alpha}_{MV} + \left(e - E\left(X_{\boldsymbol{\alpha}_{MV}}\right)\right) \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{E\left(X_{\boldsymbol{\alpha}_{SR}}\right) - E\left(X_{\mathbf{MV}}\right)}$$

$$e \in \left[E\left(X_{\boldsymbol{\alpha}_{MV}}\right), \infty\right)$$

$$\boldsymbol{\alpha}_{MV} = \frac{c Cov \mathbf{M}^{-1} d}{d' Cov \mathbf{M}^{-1} d} \qquad \boldsymbol{\alpha}_{SR} = \frac{c Cov \mathbf{M}^{-1} E(\mathbf{M})}{d' Cov \mathbf{M}^{-1} E(\mathbf{M})}$$

Two-fund separation theorem

A **linear** combination of **two** specific portfolios, $\boldsymbol{\alpha}_{MV}$ and $\boldsymbol{\alpha}_{SR}$, suffices to **generate** the whole efficient frontier

Efficient frontier with linear constraints: $c = 0$

$$\boldsymbol{\alpha}(v) = \arg\max E\left(X^{\boldsymbol{\alpha}}\right); \quad \boldsymbol{\alpha}^{'} d = 0, \;\; Var\left(X^{\boldsymbol{\alpha}}\right) = v$$

$$\boldsymbol{\alpha}(e) = e\boldsymbol{\alpha}_0$$

$$\boldsymbol{\alpha}_0 = \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{E(X_{\boldsymbol{\alpha}_{SR}}) - E(X_{\boldsymbol{\alpha}_{MV}})}$$

Mean-variance analysis can be carried out in two **alternative** frameworks involving, respectively, the **asset returns** and performance wrt a **benchmark**

Mean-variance analysis in terms of benchmark

Relevant quantities

- Absolute wealth: $X^{\boldsymbol{\alpha}} = \boldsymbol{\alpha}' \cdot \mathbf{P}_T$
- Overperformance: $\hat{X}^{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{P}_T - \gamma\boldsymbol{\beta}' \cdot \mathbf{P}_T;\ \ \gamma = \frac{\boldsymbol{\alpha}' \cdot \mathbf{P}_0}{\beta' \cdot \mathbf{P}_0}$
- Expected Overperformance: $EOP\left(\boldsymbol{\alpha}\right) = E\left(\hat{X}^{\boldsymbol{\alpha}}\right)$
- Tracking error: $TE\left(\boldsymbol{\alpha}\right) = Sd\left(\hat{X}^{\boldsymbol{\alpha}}\right)$
- Information ratio: $IP\left(\boldsymbol{\alpha}\right) = \frac{E\left(\hat{X}^{\boldsymbol{\alpha}}\right)}{Sd\left(\hat{X}^{\boldsymbol{\alpha}}\right)}$

Mean-variance analysis in terms of benchmark (continued)

$$\hat{\boldsymbol{\alpha}}(v) = \arg\max EOP(\boldsymbol{\alpha})\,; \quad \boldsymbol{\alpha}' \cdot \mathbf{P}_0 = x_0,\ \ TE^2(\boldsymbol{\alpha}) = v$$

$$\hat{\boldsymbol{\beta}} = \frac{x_0}{\mathbf{P}_0' \cdot \boldsymbol{\beta}} \boldsymbol{\beta}$$

$$\hat{X}^{\boldsymbol{\alpha}} = (\boldsymbol{\alpha} - \boldsymbol{\beta})' \cdot \mathbf{P}_T$$

$$\mathcal{C} : (\boldsymbol{\alpha} - \boldsymbol{\beta})' \cdot \mathbf{P}_T = 0$$

Defining the **relative bets** $\boldsymbol{\rho} = \boldsymbol{\alpha} - \hat{\boldsymbol{\beta}}$ the benchmark problem is reduced to the original one

$$\hat{\boldsymbol{\rho}}(v) = \arg\max E(X_{\boldsymbol{\rho}})\,; \quad \boldsymbol{\rho}' \cdot \mathbf{P}_0 = 0,\ \ Var(X_{\boldsymbol{\rho}}) = v$$

Limitations of MVT

Limitations of the mean-variance analysis

- Crucial dependence on the approximation

$$\mathcal{S}(\boldsymbol{\alpha}) \simeq \mathcal{G}\left(E\left(X^{\boldsymbol{\alpha}}\right), Var\left(X^{\boldsymbol{\alpha}}\right)\right),$$

that is valid only when one of the two cdns holds

- – the utility is **quadratic**

$$u(x) = x - \frac{1}{2\zeta}x^2$$

This is a non-intuitive utility that violates the non-saturation principle

– the market is **elliptically distributed**

$$\mathbf{M} \sim El\left(\mu, \mathbf{\Sigma}, g_N\right)$$

μ : location parameter, $\mathbf{\Sigma}$: scatter matrix,

g_N : probability generator of $N-$dim

The space of moments of the investor's objective is two-dim.

This is very strong assumption and excludes markets in which derivatives are included.

Limitations of the mean-variance analysis (continued)

- opaque dependence between the risk aversion and the market, e.g. the same investor may "display" different risk aversion depending on the market

- the dual mean-variance problem

$$\boldsymbol{\alpha}(e) = \text{arg min}\, Var\left(X_{\boldsymbol{\alpha}}\right); \;\; \boldsymbol{\alpha} \in \mathcal{C},\, E\left(X_{\boldsymbol{\alpha}}\right) \geq c$$

  might not have solutions in situations arising in Prospect Theory

- mean-variance analysis wrt returns might have serious estimation problems

- mean-variance analysis yields inconsistent results across different horizons, especially when the horizon is not close to the estimation interval

MVT and Asset Pricing

Impact of Mean-Variance analysis on Asset Pricing

- Minimum Variance portfolio (MVP)
- Efficient frontier portfolios have expected rate of return higher than the one of MVP
- For any efficient portfolio P, except the MVP, there exists its $ZC(P)$ ($Cov(P, ZC(P)) = 0$, $ZCP(ZC(P))) = P$)
- For any portfolio Q

$$E\left(r_Q\right) = \left(1 - \beta_{QZC(P)}\right) E\left(r_P\right) + \beta_{QZC(P)} E\left(r_{ZC(P)}\right)$$

$$\beta_{QP} = \frac{Cov\left(r_Q, r_P\right)}{\sigma^2(r_P)} \qquad \beta_{QZC(P)} = 1 - \beta_{QP}$$

$$\Downarrow$$

$$E\left(r_Q\right) = \beta_{QP} E\left(r_P\right) + \beta_{QZC(P)} E\left(r_{ZC(P)}\right)$$

Impact of Mean-Variance analysis on Asset Pricing (continued)

Using arguments from SSD and MV

$$\downarrow$$

$$r_Q = \beta_0 + \beta_1 r_{ZC(P)} + \beta_2 r_P + \epsilon_P$$

$$Cov\left(r_P, r_Q\right) = Cov\left(r_{ZC(P)}, \epsilon_P\right) = E\left(\epsilon_Q\right) = 0$$

$(\beta_0, \beta_1, \beta_2)$ coefficients from the multiple regression of r_Q on r_P and $r_{ZC(P)}$

$$\downarrow$$

$$\beta_0 = 0 \qquad \beta_1 = \beta_{QZC(P)} \qquad \beta_2 = \beta_{QP}$$

$$\Downarrow$$

$$r_Q = \left(1 - \beta_{QP}\right) r_{ZC(P)} + \beta_{QP} r_P + \epsilon_P$$

Impact of Mean-Variance analysis on Asset Pricing (continued)

Two-fund separation

A vector of asset returns $\mathbf{r} = \left(r_j\right)_{j=1}^{N}$ exhibits two-fund separation if there exist two mutual funds α_1 and α_2 such that for **and portfolio** Q there exists a scalar λ such that for **all** concave utilities u

$$Eu\left(\lambda\alpha_1 + (1-\lambda)\,\alpha_2\right) \geq Eu\left(r_Q\right)$$

We can easily show that the mutual funds α_1 and α_2 must be on the frontier

$$\Downarrow$$

It turns out that a **necessary and sufficient condition** for two-fund separation is

$$E\left(\epsilon_{QP}\middle|\left(1-\beta_{QP}\right)r_{ZC(P)}+\beta_{QP}\left(r_P\right)\right)=0 \quad \text{for all } Q$$

where

$$r_Q = r_{ZC(P)} + \beta_{QP}\left(r_P - r_{ZC(P)}\right) + \epsilon_{QP}$$

Impact of Mean-Variance analysis on Asset Pricing (continued)

Market Portfolio

$$W_{m0} = \sum_{i=1}^{I} W_0^i \qquad W_0^i > 0 \text{ individual wealth}$$

When two-fund separation holds then the Market Portfolio is a frontier portfolio

$$\Downarrow$$

Linear Valuation - Security Market Line (SML)

$$E\left(r_j\right) = E\left(r_{ZC(P)}\right) + \beta_{jm}\left(E\left(r_m\right) - E\left(r_{ZC(m)}\right)\right)$$

Market portfolio is efficient: $E\left(r_{ZC(m)}\right) - E\left(r_m\right) > 0$

The higher the β_{jm} for asset j, the higher its equilibrium rate of return

Capital Asset Pricing Model (CAPM)

If two-fund separation holds, risky assets are in striclty positive supply, r_f is the rate of return of the riskless asset then

$$E\left(r_Q\right) - r_f = \beta_{Qm}\left(E\left(r_m\right) - r_f\right)$$

for any portfolio Q at the market equilibrium

Lintner (1965), Mossin (1965) and Sharpe (1964)

In practice...

What happens in practice?

- Asset allocation policy that specifies **target percentages** of value for asset classes
- Analysis mainly based on MV analysis and asset pricing via $CAPM$
- Focus on the returns (unitless quantities) of candidate portfolios

Readings: W. Sharpe (2006)

Asset allocation in practice

- Selection of desired asset classes
- Selection of representative benchmark indices
- Specification of constraints/ implementation costs
- Choice of a representative historic period and specification of relevant returns of the asset classes
- Estimation of future expected returns/standard deviations/correlations (historical data, current market conditions and market interdependencies)

Asset allocation in practice (continued)

- Specification of several mean-variance efficient asset mixes for a range of risk tolerances

- Projection for future outcomes for the selected asset mixes (often for manhy years ahead)

- Presentation of the results to the board

- Choice by the board of a candidate asset mix (choice depends on the board's views of future outcomes and 'firm's risk tolerance")

Gradient Method for Portfolio Selection **(W. Sharpe: 1987, 2006)**

Assume that the only constraints are bounds on asset holdings

- Analyze an initial portfolio to find the **best** asset that could be **sold** and the best asset that could be **purchased**

- **"Best"** refers to the effect of a small change in holdings to the desirability of the portfolio to the investor

- "**Desirability"** refers to the a given target - typically expressed in terms of a quadratic criterion

Gradient Method for Portfolio Selection (continued)

- **"Desirability swap"** refers to selling and buying the appropriate assets
- Determine the **swap amount** (constraints/feasibility) so as to maximize the **increase** in portfolio desirability
- Execution of **swap transaction**
- Repetition of the process till the best swap cannot further increase the portfolio desirability

Example

Market input: States, uncertainty and future returns

States	Probability	Cash	Bond	Stock
State 1	0.25	1.05	1.0388	0.8348
State 2	0.25	1.05	0.9888	1.0848
State 3	0.25	1.05	1.0888	1.2348
State 4	0.25	1.05	1.1388	1.2848

Investor's input/objective: maximize $E\left(R_p\right) - \frac{1}{0.70} Var\left(R_p\right)$

Expected returns, standard deviations and correlation

Assets	E	SD	Cor/C	Cor/B	Cor/S
Cash	1.0500	0.0000	1.0000	0.0000	0.0000
Bond	1.0638	0.0559	0.0000	1.0000	0.6389
Stock	1.1098	0.1750	0.0000	0.6389	1.000

Mean Variance minimization

$$\Downarrow$$

Optimal asset mix

Cash	0.0705
Bond	0.3098
Stock	0.6196

Portfolio construction

- Build the efficient frontier (this requires knowledge of returns/covariance/risk tolerance)

- Determine the risk free rate r_f

- Draw the tangent to the frontier. The point it touches the frontier yields the asset class mix with the highest Sharpe ratio

Portfolio construction (continued)

- Lowering the risk tolerance will result in including more cash but the allocation mix (bonds-stocks) will remain the same

- Increasing your risk tolerance to a high enough level will yield a zero-cash portfolio. This means you're up on the Efficient Frontier, but to the right of the point where it intersects the straight line. (In theory you could get up to the line even here if you are willing to hold a "negative" amount of cash, that is, to invest on margin.)

- Decreasing the covariance between stocks and bonds, will result in allocating more money to stocks and bonds and less to cash, thus raising the rate of return

Optimal investments and index funds

- In theory there are many assets in the portfolio. Impossible to implement in practice
- Need to use the so called index funds
- Efficient market hypothesis trusting the market to price the representative indices

Tobin's Separation Theorem

The optimal investment problem can be solved as follows: first find the optimal combination of risky securities and then deciding whether to lend or borrow, depending on your attitude toward risk. If there is only one portfolio plus borrowing and lending, this has to be the market portfolio.

Thus all optimal portfolios are made with one optimal securities mix plus a varying amount of cash

Calculating equilibrium asset returns

$$r = Rf + beta(r_M - rf)$$

- r is the expected return rate on a security
- r_f is the rate of a "risk-free" investment, i.e. cash
- r_M is the return rate of the appropriate asset class

Beta measures the volatility of the security, relative to the asset class

- Investors require higher levels of expected returns to compensate them for higher expected risk
- Knowledge of the security's beta yields the value of r that investors expect it to have

Calculating equilibrium asset returns (continued)

Questions

- What security classes should we use?
- Coarse (stocks-bonds) or finer (domestic mid-cap etc)
- What should beta reflect (status of compnay, its debt etc)

Consequences of CAPM in investment practice

- Finding the efficient frontier is feasibe, because one only has to calculate the covariance matrix of the assets in the appropriate class
- Individual stocks in a certain class can be replaced by their representative index
- Beating the index almost impossible (but doable) due to fees and other frictions
- Beating the performance of an asset class requires negative cash contribution, i.e. buying the index on the margin (very risky)
- How well can fund managers do after all?

Alpha and beta indices

$$r - r_f = beta(r_M - r_f) + alpha$$

Alpha, the vertical intercept, expresses **how much better** the fund did than CAPM predicted **Factor models**

Three factor model (G. Fama and K. French)

Key observation: two classes of stocks frequently do better than the market as a whole: **small caps** and **value stocks** (stocks with a high book-value-to-price ratio— their opposites are growth stocks)

Alpha and beta indices (continued)

$$r - r_f = beta_3(r_M - r_f) + (b_s)\,SMB + (b_v)\,HML + alpha$$

- r_m is the return of the entire stock market
- SMB small cap minus big
- HML high (book/ price) minus low
- b_s (~ 1 corresponds to mainly small cap portfolio)
- b_v ($\sim$ 1 corresponds to a portfolio with a high book/priceratio)

Expected utility asset allocation (W. Sharpe: 2006)

- Maximize expected utility of a risk-averse investor

$$Eu = \sum_s \pi_s u(R_{ps})$$

Marginal expected utility (per dollar) of individual asset i

Utility is asssumed to be the same for all states

- Constraints

$$lb_i \leq x_i \leq ub_i \qquad (b: \text{bound}, \ x: \text{holding}, \ l, u: \text{lower/upper})$$

- Marginal expected utility of the portfolio return for a given state s

$$mEu\left(R_{ps}\right) = \pi_s m\left(R_{ps}\right)$$

- Marginal expected utility (per dollar) of individual asset i

$$mEu_i = \sum_s R_{is} mEu\left(R_{ps}\right) = \sum_s R_{is} \pi_s m\left(R_{ps}\right)$$

- Desirability swaps whenever $mEu_i > mEu_j$

Expected utility asset allocation (continued)

- Compute marginal expected utility of individual assets
- Classify assets as potential buys (if $x_i < ub_i$) or potential sell (if $x_i > lb_i$)
- Find the "best buy", b^*, i.e. the asset among potential buys with the largest marginal expected utility
- Find the "best sell", s^*, i.e. the the asset among potential sells with the smallest marginal expected utility

- If $mEu_{b^*} > mEu_{s^*}$ the best swap involves selling units of the best sell and purchasing units of the best buy (If this is not the case, or there no potential buys or sells, the portfolio cannot be improved

- Determine the optimal swap amount

- Revise portfolio

- Continue till no desirability swap exists