Life-cycle housing and portfolio choice with bond markets*

Otto van Hemert
Swedish Institute for Financial Research, University of Amsterdam

November 20, 2005

Abstract

I study optimal housing and portfolio choice under stochastic inflation and real interest rates. Renters allocate financial wealth to stocks and bonds with different maturities. Homeowners also choose the mortgage type. I show that hedge demands and financial constraints vary over an investor's lifetime, giving rise to a pronounced life-cycle pattern in the optimal housing, stock, bond, and mortgage choice. Young homeowners take an adjustable-rate mortgage (ARM) and invest financial wealth predominantly in stocks. Later in the life cycle bonds play an important role, mainly as a hedge against changing real interest rates and house prices. Fairly risk-tolerant homeowners still prefer an ARM, while more risk-averse investors rather choose a combination of an ARM and a fixed-rate mortgage.

*This paper has benefited from discussions with Ulf Axelson, Magnus Dahlquist, Frank de Jong, Joost Driessen, Enrico Perotti, Per Strömberg and David Yermack. In addition, discussions with João Cocco, Francisco Gomez, Alex Michaelides and Yihong Xia on a previous, related, paper were helpful. I would like to thank SARA for the use of their supercomputer. Contact address: Swedish Institute for Financial Research, Saltmätargatan 19 A, SE-113 59 Stockholm, Sweden. E-mail: otto.van.hemert@sifr.org.

1 Introduction

For many investors housing is the largest asset, and the mortgage on the house the largest liability. Both are likely to have a major impact on the optimal financial portfolio choice. This paper shows that besides stocks, bonds and mortgages play an important role in a homeowner's financial portfolio. Together, the bonds and the mortgage determine the duration of the overall portfolio, which is important for hedging real interest rate risk. In addition, the bonds and the mortgage may provide a partial hedge against house price changes. I show that hedge demands and financial constraints vary over an investor's lifetime, giving rise to a pronounced life-cycle pattern in the optimal housing, stock, bond, and mortgage choice.

This paper basically merges two recent strands in the portfolio choice literature. Papers in the first strand investigate life-cycle portfolio choice while taking into account the role of housing. Cocco (2005) investigates the joint decision on owner-occupied housing and portfolio choice. Yao and Zhang (2005) also include a choice on housing tenure.[1] Both papers restrict the asset menu to stocks and cash, and do not consider a mortgage choice. I extend these papers by adding bonds with different maturities to the asset menu, studying mortgage choice, and modelling the interaction of the housing return with financial asset returns in a more sophisticated manner. The second strand illustrates the importance of bonds for a long-term investor. Examples are Brennan and Xia (2002) and Campbell and Viceira (2001). Both papers use a two-factor model similar to mine for the nominal interest rate. A long-term investor holds bonds not only to exploit the risk premium, but also to hedge changes in the investment opportunity set. My paper extends these papers to a life-cycle setting with risky housing and labor income. In addition, I take into account the housing tenure, house size and mortgage choice.

I consider the following model. An investor receives stochastic, exogenous labor income until retirement and derives utility from both housing and other goods consumption. Investors dynamically decide on their housing tenure, house size, financial portfolio and other goods consumption. For homeowners the house not only provides housing services, but also entails a risky investment. An investor can change his tenure and house size only at a transaction cost. This cost is larger when moving to an owner-occupied house than to a rental house. Renters choose how to allocate financial wealth to stocks, bonds with

[1] Another strand of literature focuses on the housing and financial portfolio choice in a static one-period mean-variance setting. Examples are Brueckner (1997) and Flavin and Yamashita (2002).

different maturities, and cash. Financial positions can be adjusted without transaction cost. Negative positions are precluded. Homeowners also choose the mortgage type and size. A homeowner may take a mortgage loan up to the market value of the house minus a down payment. I allow for an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a combination of the two (hybrid mortgage). A homeowner can adjust his mortgage type and size at zero cost, as is typically the case for a home line of credit. The ARM is modelled as a negative cash position, and the FRM as a negative position in a long-term bond.

For the asset price dynamics I extend the Brennan and Xia (2002) model with a house price and labor income process. Nominal bonds are priced by a two-factor model for the term structure of interest rates. I use expected inflation and real interest rate as factors. In contrast to a one-factor model, this model provides a rationale for holding nominal bonds with different maturities. Importantly, it also allows me to investigate the implications of different types of mortgages. I also model unexpected inflation, house price risk, labor income risk and stock market risk, leading to a total of six sources of uncertainty. This structure enables me to realistically examine the interaction of different asset prices. The parameter values for these price dynamics are largely based on estimates by Van Hemert, De Jong and Driessen (2005), who use US data. In accordance with other papers,[2] they find a faster mean-reversion in the real interest rate than in the expected inflation rate. This turns out to be crucial for the choice among bonds with different maturities and the mortgage choice.

The main results of the paper can be summarized as follows. The motivation to hold risky assets varies over an investor's lifetime, giving rise to a clear life-cycle pattern in his optimal house, stock, bond and mortgage choice. An investor starts adult life with little financial wealth and large human capital, making him severely borrowing constrained. The investor rents the house he lives in and only holds a small buffer capital as financial wealth. Over time more labor income is earned and the investor starts to save for the down payment on an owner-occupied house. In this period he becomes less borrowing constrained, but is still very short-sale constrained. Taking into account his large human capital, the investor chooses an almost 100% stock allocation in order to exploit the equity premium, which is set at 4% in my analysis.

Per-period costs for a given house size are smaller when owning than when renting. This makes the investor so eager to buy his first house that the move from a rental to an

[2] See e.g. Brennan and Xia (2002) or Campbell and Viceira (2001).

owner-occupied house often involves moving to a smaller house, for which he is just able to pay the required down payment. The young homeowner optimally chooses an ARM of maximum size, irrespective of risk aversion. This allows a homeowner to exploit the risk premium on stocks and bonds.

As a homeowner builds up more financial wealth, he typically decides to move to a bigger owner-occupied house. With the larger physical (financial plus housing) capital and smaller human capital, the desire to take risk and exploit risk premia decreases, while the desire to hedge against changing investment opportunities becomes more important. Initially a homeowner chooses a long-term bond for this hedge, because it has the largest absolute loading on real interest rate risk. When approaching retirement age the allocation starts to shift towards short-term bonds, which have the larger loading on real interest rate risk relative to expected inflation risk. A fairly aggressive homeowner will still hold a considerable amount of long-term bonds and stocks at retirement. A more risk-averse homeowner, who is more concerned with hedging real interest rate risk, will almost completely shift to short-term bonds. Moreover, to further improve the effectiveness of the real interest rate hedge, he desires to short-sell the long-term bond. The optimal mortgage for this more risk-averse homeowner consequently changes from a pure ARM to a hybrid mortgage, modelled as a short position in both cash and a long-term bond.

Towards the end of his lifetime the investor sells his house and starts renting again. This enables him to consume all his wealth, including the down payment on the previously owned house. In anticipation of this sell, the investor adjusts his financial portfolio to hedge against house price falls.

In addition to the above-mentioned papers, this paper also relates to Campbell and Cocco (2003). In this paper the choice between an FRM and an ARM involves a trade off between what they refer to as wealth and income risk. An FRM has a variable real value, leading to wealth risk. An ARM has an almost fixed real value, but has, in their set up, short-term variability in real payments, leading to income risk. My mortgage analysis differs from Campbell and Cocco (2003) in several important ways. Campbell and Cocco (2003) do not consider stocks and bonds, and assume all other financial wealth is invested in cash. In contrast, I consider mortgage choice as part of the overall financial portfolio choice. While Campbell and Cocco (2003) incorporate persistent shocks to the expected inflation only, I allow for persistent shocks in the real interest rate as well. Together the bonds and mortgage determine the duration of the overall portfolio, which is important for hedging real interest rate risk. Even though there is no income risk of the above kind in my model, these considerations make the choice between an ARM and an FRM interesting in my set up. Moreover, in contrast to Campbell and Cocco (2003), I allow for a tenure and house size choice, which enables me to study mortgage choice in a broader context.

Finally this paper relates to Van Hemert, De Jong and Driessen (2005), who study a homeowner's optimal portfolio choice assuming (i) utility of terminal wealth, (ii) no labor income, (iii) fixed housing investment. Similar to this paper, they use a two-factor model to decribe bond prices and model an ARM (FRM) as a short position in cash (a long-term bond). In contrast to Van Hemert, De Jong and Driessen (2005), I use a life-cycle setting with stochastic labor income and find a pronounced life-cylce pattern in optimal choices. Moreover, I allow for a tenure and house size choice.

The structure of this paper is as follows. Section 2 presents the model. Section 3 discusses the estimation of the model parameters. Section 4 contains the main results and section 5 concludes.

2 The economic model

I study optimal financial planning for an investor from time 0 to time $T = 60$ years, corresponding to age 20 to 80. I abstract from longevity risk. The investor faces a choice regarding (i) tenure, (ii) house size, (iii) allocation of financial assets, including a choice on mortgage type, and (iv) consumption. I interpreted house size as a one-dimensional representation of the quality of the house. Each period the investor can either stay in the same (size) house or move and pay some transaction costs. During working life the investor receives labor income which I assume is exogenous.[3]

2.1 Preferences

The investor derives utility from both the housing services and other goods consumption, c. The real price of consumption goods is chosen to be the numeraire and the real price of a unit of housing is denoted q (with $q_0 \equiv 1$). I denote the house size at time t by H_t. Following Cocco (2005), Hu (2003) and Yao and Zhang (2005) I represent preferences over

[3] Bodie, Merton and Samuelson (1992) show that endogenous labor income may increase the optimal risk taking in the financial portfolio.

housing consumption to other goods by the Cobb-Douglas function

$$U_t = \int_t^T \beta^{s-t} \frac{\left(c_s^{1-\psi} H_s^{\psi}\right)^{1-\gamma}}{1-\gamma} ds \tag{1}$$

where U_t is lifetime utility, β is the subjective discount rate, γ is the coefficient of relative risk aversion, and ψ is the relative preference for housing consumption.

2.2 Asset price dynamics

I consider an economy with six sources of uncertainty represented by innovations in six Brownian motions. I assume the investor takes price processes as given. Furthermore, I assume that the risk premia on the sources of uncertainty are constant. Financial asset, mortgage and house prices are determined by the dynamics of the first five sources of uncertainty. For this I use the setup of Van Hemert, De Jong and Driessen (2005) who in turn, extend Brennan and Xia (2002) with an additional source of uncertainty to capture house price risk. The five variables that determine asset prices are: nominal stock return S, instantaneous real interest rate r, instantaneous expected inflation rate π, nominal house price Q, and the price level Π. The equations driving these variables are given by

$$\begin{aligned}
dS/S &= \left[R_f + \sigma_S \lambda_S\right] dt + \sigma_S dz_S, && (2)\\
dr &= \kappa \left(\bar{r} - r\right) dt + \sigma_r dz_r, && (3)\\
d\pi &= \alpha \left(\bar{\pi} - \pi\right) dt + \sigma_\pi dz_\pi, && (4)\\
dQ/Q &= \left[R_f + \sigma_Q \lambda_Q - r^{imp}\right] dt + \sigma_Q dz_Q, && (5)\\
d\Pi/\Pi &= \pi dt + \sigma_\Pi dz_\Pi, && (6)
\end{aligned}$$

where R_f is the return on the nominal risk free asset (cash), λ_S and λ_Q are nominal risk premia, r^{imp} is the imputed rent, dz's are standard Brownian motions and the σ's capture the volatility of the processes. The risk premium on the house is corrected by the imputed rent, representing the benefits from the housing services (as measured by the market). We can orthogonalize (5) and (6) as

$$\begin{aligned}
dQ/Q &= \left(R_f + \theta' \lambda - r^{imp}\right) dt + \theta' dz, && (7)\\
d\Pi/\Pi &= \pi dt + \xi' dz + \xi_u dz_u, && (8)
\end{aligned}$$

with $\theta = (\theta_S, \theta_r, \theta_\pi, \theta_v)'$, $\xi = (\xi_S, \xi_r, \xi_\pi, \xi_v)'$, $\lambda = (\lambda_S, \lambda_r, \lambda_\pi, \lambda_v)'$ and $z = (z_S, z_r, z_\pi, z_v)$, where dz_v is orthogonal to dz_S, dz_r and dz_π and dz_u is orthogonal to dz. Defining the covariance matrix of dz by

$$\rho = \begin{pmatrix} \rho_{S,r,\pi} & 0 \\ 0 & 1 \end{pmatrix}, \tag{9}$$

we have $\sigma_Q^2 = \theta' \rho \theta$ and $\sigma_\Pi^2 = \xi' \rho \xi + \xi_u^2$.

Brennan and Xia (2002) show that the nominal price at time t of a discount bond with a \$1 nominal payoff maturing at time T, denoted as P, satisfies

$$dP/P = (R_f - B\sigma_r\lambda_r - C\sigma_\pi\lambda_\pi)\, dt - B\sigma_r dz_r - C\sigma_\pi dz_\pi, \tag{10}$$

where $B(T-t) = \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right)$, and $C(T-t) = \alpha^{-1}\left(1 - e^{-\alpha(T-t)}\right)$ are functions of the time to maturity $T-t$. The return processes for bonds with different maturities differ only in their loadings on dz_r and dz_π. When there are no constraints on position size, any desired combination of loadings on dz_r and dz_π can be accomplished by positions in any two bonds with different maturities.

Real returns for stocks, bonds and the house can easily be obtained using (8) and applying Ito's lemma. I use uppercase letters for nominal variables and the corresponding small case letter for their real counterpart. We have $R_f = r + \pi - \xi'\lambda - \xi_u\lambda_u$ and for example the real return on stocks is given by

$$ds/s = \left(r + \sigma_S\lambda_S^* - \xi'\lambda^* - \xi_u\lambda_u^*\right) dt + \sigma_S dz_S - \xi' dz - \xi_u dz_u. \tag{11}$$

where the $*$ refers to a real risk premia, i.e. $\lambda^* = \lambda - \rho\xi$ and $\lambda_u^* = \lambda_u - \xi_u$.

2.3 Investment Opportunity Set

The menu of available financial assets consists of stocks, 3-year bonds, 10-year bonds and cash. The two bonds are assumed to be zero-coupon bonds. I impose short-sale constraints on these assets. In addition, for homeowners the house not only provides housing services, but also entails a (risky) investment. Notice in equation (11) that the real risk premium for stocks is fixed and in particular independent of the expected inflation rate, π. The same holds for the real risk premium on the house, nominal bonds and cash, which implies that the real investment opportunity set in my model is independent of the prevailing expected inflation rate.

An investor cannot borrow against his human capital. However, homeowners can take a mortgage loan up to a fraction $1-\delta$ of the market value of the house, where δ is the minimum down payment fraction. He can use the proceeds to consume or to invest in stocks, bonds or cash. I include the (negative) market value of the mortgage in my definition of financial wealth, which therefore can become negative. Total (financial plus housing) wealth, however, cannot be less than the minimum down payment of δ times the value of the house. Following Cocco (2005) I assume that a homeowner can costlessly adjust the mortgage, as is typically the case for a home line of credit.

A homeowner can choose between an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a hybrid mortgage which is a combination of an ARM and an FRM. I model an ARM (FRM) as a short position in cash (10-year bond), i.e. I assume the (relative) increase in the market value of the loan equals the return on cash (10-year bond). Since I also allow for hybrid mortgages, the investor basically can take a negative cash and 10-year bond position, each and added up not to exceed $(1-\delta)$ times the market value of the house.

2.4 Labor income

The sixth source of uncertainty captures labor income risk. Real labor income, l, is assumed to be subject to permanent shocks only.[4] In addition, real labor income has a deterministic component $g(t)\,dt$ that captures the hump-shaped pattern of labor income

$$
\begin{aligned}
dl/l &= g(t)\,dt + \sigma_l dz_l \quad \text{for } t \leq 45 \qquad (12)\\
l &= 0 \qquad\qquad\qquad\;\; \text{for } t > 45 \qquad (13)
\end{aligned}
$$

I assume that dz_l is orthogonal to dz and dz_u, which implies that labor income shocks are completely unhedgeable.

After retirement at time 45 (age 65) labor income is assumed to be zero. I have experimented with different assumptions regarding pension income and bequest motives and found that *with* substantial pension income and *without* bequest motive investors save unrealistically little in my Cobb-Dougles utility framework. Cocco (2005) and Yao and Zhang (2005) have pension income and a bequest motive. However, the empirical evidence for the

[4] Viceira (2001), Yao and Zhang (2003) and Munk and Sørensen (2005) also assume stochastic shocks to permanent labor income only. Cocco, Gomes and Maenhout (2005), Campbell and Cocco (2003) and Cocco (2005) also allow for transitory, individual labor income shocks.

latter is mixed; see for example Hurd (1989) and Bernheim (1991) for a positive and negative view on a strong bequest motive respectively. In contrast, I choose to report results for the no pension income and no bequest motive case, and therefore study a investor who saves for his own retirement and eventually consumes the (retirement) wealth he accumulated. Since I abstract from longevity risk, the investor is able to exhaust his savings fully.

2.5 Housing costs

Renters pay a fixed fraction ζ of the market value of the house as rent. Homeowners have to pay maintenance costs equal to a fraction m of the market value of the house. Typically I will have $\zeta > m$, implying that the per period per unit housing costs are lower for homeowners. When moving from one house to another a one-time transaction cost is incurred. I assume this cost is equal to a fraction ν^{rent} (ν^{own}) of the market value of the new house when he rents (owns) the new house. We will have $\nu^{own} > \nu^{rent}$.

2.6 Optimization problem

I denote real housing wealth by w^H and real financial wealth by w^F. I define the indicator variable I as being one (zero) when the investor is currently owning (renting). Total real wealth, w, is then given by

$$w = w^F + Iw^H \tag{14}$$

For homeowners financial wealth includes the mortgage and can be negative. The evolution of real financial and housing wealth during periods with no moving is given by

$$\begin{aligned} dw^F &= \left[\left\{r + \mu_F^e(x) - \xi'\lambda^* - \xi_u\lambda_u^*\right\} w^F - \left\{(1-I)\zeta + Im\right\} w^H + l - c\right] dt + \qquad (15) \\ &\quad \sigma_F'(x)\, w^F dz - \xi' w^F dz - \xi_u w^F dz_u, \\ dw^H &= \left\{r + \mu_H^e - \xi'\lambda^* - \xi_u\lambda_u^*\right\} w^H dt + \qquad (16) \\ &\quad \sigma_q' w^H dz - \xi' w^H dz - \xi_u w^H dz_u, \end{aligned}$$

where μ_F^e and μ_H^e are the excess nominal return on financial and housing wealth respectively, and σ_F and σ_q are vectors containing the factor loadings on dz for the nominal return on financial and housing wealth respectively. We have $\mu_H^e = \theta'\lambda^*$ and $\sigma_q = \theta$. We have that μ_F^e and σ_F are independent of w^F and w^H, but they do depend on the chosen portfolio shares, denoted by $x = \left(x^{stocks}, x^{3ybond}, x^{10ybond}, x^{cash}\right)$, in the following manner

$$\begin{aligned} \mu_F^e(x) &= x^{stocks}\sigma_S\lambda_S^* + \\ &\quad \left[-x^{3ybond}B(3) - x^{10ybond}B(10)\right]\sigma_r\lambda_r^* + \\ &\quad \left[-x^{3ybond}C(3) - x^{10ybond}C(10)\right]\sigma_\pi\lambda_\pi^*, \end{aligned} \tag{17}$$

$$\sigma_F(x) = \begin{pmatrix} x^{stocks}\sigma_S \\ \left[-x^{3ybond}B(3) - x^{10ybond}B(10)\right]\sigma_r \\ \left[-x^{3ybond}C(3) - x^{10ybond}C(10)\right]\sigma_\pi \\ 0 \end{pmatrix}. \tag{18}$$

When the investor moves at time t and the new tenure state and house size are denoted by I_t^{new} and H_t^{new} respectively, the change in housing wealth is given by $dw_t^H = q_t\left(H_t^{new} - H_t\right)$. Denoting $w^{H,new} \equiv q_t H_t^{new}$, the change in financial wealth is given by

$$dw_t^F = -\nu^{rent}w_t^{H,new} \qquad \text{for } I_t = 0,\ I_t^{new} = 0 \text{ (rent to rent)} \tag{19}$$

$$dw_t^F = -\nu^{own}w_t^{H,new} - w_t^{H,new} \qquad \text{for } I_t = 0,\ I_t^{new} = 1 \text{ (rent to own)} \tag{20}$$

$$dw_t^F = -\nu^{rent}w_t^{H,new} + w_t^H \qquad \text{for } I_t = 1,\ I_t^{new} = 0 \text{ (own to rent)} \tag{21}$$

$$dw_t^F = -\nu^{own}w_t^{H,new} - w_t^{H,new} + w_t^H \qquad \text{for } I_t = 1,\ I_t^{new} = 1 \text{ (own to own)} \tag{22}$$

The state variables for the investor's investment problem are given by I, w^F, w^H, l, q, r and t. From equations (15)-(22) it is clear that a housing and other goods consumption strategy $\{c_t, H_t\}_t^T$ is sustainable starting in state $\left(I, w^F, w^H, l, q, r, t\right)$ if and only if the consumption strategy $\{\tau c_t, \tau H_t\}_t^T$ is sustainable starting in state $\left(I, \tau w^F, \tau w^H, \tau l, q, r, t\right)$ for any $\tau > 0$. From equation (1) we see that the lifetime utility function, U_t, is homogeneous of degree $1-\gamma$ in $\{c_t, H_t\}_t^T$, allowing us to simplify indirect utility as

$$J\left(w^F, w^H, l, q, r, t\right) = w^{1-\gamma}J\left(1 - Ih, h, y, q, r, t\right). \tag{23}$$

where $h = w^H/w$ and $y = w/l$. Similarly, from equations (15)-(22) it is clear that consumption strategy $\{c_t, H_t\}_t^T$ is sustainable starting in state $\left(I, w^F, w^H, l, q, r, t\right)$ if and only if the consumption strategy $\{c_t, \tau H_t\}_t^T$ is sustainable starting in state $\left(I, w^F, w^H, l, q/\tau, r, t\right)$ for any $\tau > 0$. Exploiting that the lifetime utility function, U_t, is homogeneous of degree

$(1-\gamma)\,\psi$ in $\{H_t\}_t^T$, I can further simplify it as

$$J\left(w^F, w^H, l, q, r, t\right) = \left(\frac{w}{q^\psi}\right)^{1-\gamma} J\left(I, 1-Ih, h, y, 1, r, t\right) \tag{24}$$

$$\equiv \left(\frac{w}{q^\psi}\right)^{1-\gamma} \tilde{J}\left(I, y, h, r, t\right). \tag{25}$$

So two state variables are separable, which greatly helps for solving for the optimal strategy.

2.7 Solution technique

Given the finite nature of the problem a solution exists. We have that $\tilde{J}\left(I_T, y_T, h_T, r_T, T\right) = 0$, since the investor derives no utility from leaving a bequest. A grid over y, h, r and t is chosen to numerically solve for $\tilde{J}\left(I_t, y_t, h_t, r_t, t\right)$ and the optimal choices I_t^{new}, H_t^{new}, c_t and x_t backwards in time. Without loss of generality we normalize $w_t = 1$ and $q_t = 1$ in each step. Thus I determine $\tilde{J}\left(I_t, y_t, h_t, r_t, t\right)$ by solving

$$\begin{aligned} \tilde{J}\left(I_t, y_t, h_t, r_t, t\right) &= \max_{I_t^{new}, H_t^{new}, c_t, x_t} \frac{\left(c_t^{1-\psi}\left(H_t^{new}\right)^\psi\right)^{1-\gamma}}{1-\gamma}\Delta t \\ &\quad + E\left[\beta^{\Delta t}\left(\frac{w_{t+\Delta t}}{q_{t+\Delta t}^\psi}\right)^{1-\gamma}\tilde{J}\left(I_{t+\Delta t}, y_{t+\Delta t}, h_{t+\Delta t}, r_{t+\Delta t}, t+dt\right)|\mathcal{F}\right] \qquad (26) \\ \text{with } \mathcal{F} &= \left\{w_t = 1, q_t = 1, y_t, h_t, r_t, t\right\} \end{aligned}$$

where Δt is the step size of the grid over time. To determine $\tilde{J}\left(I_{t+\Delta t}, y_{t+\Delta t}, h_{t+\Delta t}, r_{t+\Delta t}, t+\Delta t\right)$ for values of $y_{t+\Delta t}$, $h_{t+\Delta t}$ and $r_{t+\Delta t}$ not on the grid, I use linear interpolation on $\log(\tilde{J})$. The expectation is evaluated using a 3-point Gaussian quadrature for each of the six sources of uncertainty represented by the six Brownian motions. For the optimization I use the Downhill Simplex Method in Multidimensions (Nelder and Mead (1965)) which doesn't use any derivative information and is robust to different starting values. The chosen timing of events in each period is the following.

1. The investor starts period t with normalised total wealth of $w_t = 1$ and house price $q_t = 1$. The state variables y_t and h_t determine the labor income rate $l_t = w_t/y_t$, and the current house size $H_t = w_t h_t/q_t$ respectively.

2. The investor chooses the new tenure state, I_t^{new}, and house size H_t^{new}. When $H_t \neq H_t^{new}$ or $I_t \neq I_t^{new}$, the investor chose to move and transaction costs will be incurred.

Irrespective of whether the investor moved or not, rental or maintenance cost are incurred. The total housing costs, denoted $costs_t$, are given by

$$\begin{aligned} costs_t &= (1 - I^{new})\,\zeta H^{new} q_t \Delta_t + I^{new} m H^{new} q_t \Delta_t + \\ &\quad Ind\{move\}\left[(1 - I_t^{new})\,\nu^{rent} H_t^{new} q_t + I_t^{new} \nu^{own} H_t^{new} q_t\right], \end{aligned} \tag{27}$$

where $Ind\{move\}$ is one (zero) when the investor moved (did not move).

3. The investor chooses consumption c_t, such that the total wealth after consumption and housing costs, $\hat{w}_t$, is at least zero for a renter or the compulsory down payment on the house for a homeowner,

$$\hat{w}_t = w_t - c_t \Delta t - costs_t \geq I_t^{new} \delta H_t^{new} q_t. \tag{28}$$

4. Then the allocation at time t over stocks ($stock_t$), 3-year bond ($3ybond_t$), 10-year bond ($10ybond_t$) and cash ($cash_t$) is chosen under the following constraints

$$0 \leq stock_t \leq \hat{w}_t - I_t^{new} \delta H_t^{new} q_t \tag{29}$$

$$0 \leq 3ybond_t \leq \hat{w}_t - I_t^{new} \delta H_t^{new} q_t \tag{30}$$

$$I_t^{enw}(1 - \delta) H_t^{new} q_t \leq 10ybond_t \leq \hat{w}_t - I_t^{new} \delta H_t^{new} q_t \tag{31}$$

$$cash_t = \hat{w}_t - stock_t - 3ybond_t - 10ybond_t - I_t^{new} H_t^{new} q_t \tag{32}$$

Since a homeowner can take a mortgage of only a fraction $(1 - \delta)$ of the house value, his risky asset allocation cannot exceed $\hat{w}_t - \delta H_t^{new} q_t$.

5. He earns a return on the financial assets and receives labor income resulting in

$$w_{t+\Delta t} = cash_{t+\Delta t} + stock_{t+\Delta t} + 3ybond_{t+\Delta t} + 10ybond_{t+\Delta t} + \tag{33}$$

$$I_t^{new} H_t^{new} q_{t+\Delta t} + l_{t+\Delta t} \Delta t \tag{34}$$

$$I_{t+\Delta t} = I_t^{new} \tag{35}$$

$$y_{t+\Delta t} = w_{t+\Delta t} / l_{t+\Delta t} \tag{36}$$

$$h_{t+\Delta t} = H_t^{new} q_{t+\Delta t} / w_{t+\Delta t} \tag{37}$$

I choose $\Delta t = 1/12$, i.e. a month. For y I choose a 60 point grid on $[\Delta t, 20]$. For h I choose an equally-spaced grid on $[0, 4]$ with step size 0.1.

3 Calibration

The parameter values for the asset price dynamics and labor income process are presented in Table I. The values for the real riskless rate, expected inflation rate and unexpected inflation rate are mostly taken from Van Hemert, De Jong and Driessen (2005). They use quarterly data on US nominal interest rates and inflation from 1973Q1 to 2003Q4 and employ a Kalman filter technique. For more details I refer to Van Hemert, De Jong and Driessen (2005). A faster mean reversion in the real interest rate than in the expected inflation rate is in accordance with e.g. Brennan and Xia (2002) and Campbell and Viceira (2001). In addition to the parameters provided by Van Hemert, De Jong and Driessen (2005), I set the nominal unexpected inflation premium, λ_u, equal to zero.

Also similar to Van Hemert, De Jong and Driessen (2005), parameter values for stock and house price dynamics are based on quarterly US data from 1980Q2 to 2003Q4. For the stock data I use an index comprising all NYSE, AMEX and NASDAQ firms.[5] For house price data I use a repeated sales index for houses in Atlanta, Boston, Chicago and San Francisco.[6] I have no data on market imputed rent, but for the financial asset allocation $\theta_v \lambda_v - r^{imp}$ and not λ_v and r^{imp} seperately is relevant. I can estimate $\theta_v \lambda_v - r^{imp}$ from the data and simply set r^{imp} equal to the mean real interest rate $\bar{r}$. As Van Hemert, De Jong and Driessen (2005) I scale house price shocks with a factor 5.6 around its mean to reflect the fact that house prices are subject to idiosyncratic shocks in addition to aggregate shocks. In contrast to Van Hemert, De Jong and Driessen (2005), here I calculate correlations with house price innovation on a yearly instead of a quarterly basis. I choose to do so because house prices may adjust slower to news than financial assets. Extending the calibration horizon beyond one year makes little difference. Nominal house price changes are found to be negatively correlated with real interest rate shocks and positively correlated with expected inflation shocks. The scaling of house prices might lead to coefficients of correlation with financial asset prices that are biased upwards in size. As robustness check I investigate model outcomes with the alternative assumption of correlations between housing and financial assets equal to zero.

I consider a horizon of $T = 60$ years, corresponding to age 20 to 80. The investor is assumed to retire at time 45 or age 65. I follow Munk and Sørensen (2005) by adapting the estimated labor income profile of Cocco, Gomes and Maenhout (2005) to a continuous-time

[5] I would like to thank Kenneth R. French for making this data available at his website.

[6] I would like to thank the Case-Shiller-Weiss company for providing us with this data.

Table I. Choice of asset price parameters.
The table reports calibrated values for the parameters that drive asset price and labor income dynamics.

Parameter	Estimate	(Alternative)
Stock return process: $dS/S = (R_f + \sigma_S \lambda_S)\, dt + \sigma_S dz_S$		
σ_S	0.1748	
λ_S	0.2288	
Real riskless interest rate process: $dr = \kappa\,(\bar{r} - r)\, dt + \sigma_r dz_r$		
$\bar{r}$	0.0226	
κ	0.6501	
σ_r	0.0183	
λ_r	-0.3035	
Expected inflation process: $d\pi = \alpha\,(\bar{\pi} - \pi)\, dt + \sigma_\pi dz_\pi$		
$\bar{\pi}$	0.0351	
α	0.0548	
σ_π	0.0191	
λ_π	-0.1674	
House price process: $dQ/Q = \left(R_f + \sigma_Q \lambda_Q - r^{imp}\right) dt + \sigma_Q dz_Q = \left(R_f + \theta' \lambda - r^{imp}\right) dt + \theta' dz$		
θ_S	0.0079	(0.0000)
θ_r	-0.0129	(0.0000)
θ_π	0.0427	(0.0000)
θ_v	0.1418	(0.1500)
λ_v	0.1315	(0.1150)
r^{imp}	0.0226	
σ_Q	0.1500	
λ_Q	0.1150	
Realized inflation process: $d\Pi/\Pi = \pi dt + \sigma_\Pi dz_\Pi = \pi dt + \xi' dz + \xi_u dz_u$		
ξ_S	-0.0033	
ξ_r	0.0067	
ξ_π	0.0012	
ξ_v	-0.0236	(0.0000)
ξ_u	0.0474	(0.0530)
λ_u	0.0000	
σ_Π	0.0535	
Real labor income process: $dl/l = g\,(t)\, dt + \sigma_l dz_l$, where $g\,(t) = b + c\,(t+20) + 3d\,(t+20)^2$		
σ_l	0.1000	
b	0.1682	
c	-0.00323	
d	0.000020	
Correlations:		
ρ_{Sr}	-0.1643	
$\rho_{S\pi}$	0.0544	
$\rho_{r\pi}$	-0.2323	

Table II Correlation matrix for $(dz_S, dz_r, dz_\pi, dz_Q, dz_\Pi, dz_l)'$

	dz_S	dz_r	dz_π	dz_Q	dz_Π	dz_l
dz_s	1					
dz_r	-0.1643	1				
dz_π	0.0544	-0.2323	1			
dz_Q	0.0826	-0.1608	0.3075	1		
dz_Π	-0.0809	0.1294	-0.0090	-0.4355	1	
dz_l	0	0	0	0	0	1

setting. The deterministic part of the change in labor income is given by

$$g(t) = b + 2c(t+20) + 3d(t+20)^2. \tag{38}$$

where $t+20$ is the age. Cocco, Gomes and Maenhout (2005) estimate b, c and d for three groups characterized by the highest level of education achieved: "No High school", "High school" and "College". I focus on the "High school" group. I follow Munk and Sørensen (2005) and set the income rate volatility at $\sigma_l = 0.10$. Recall that after retirement (at age 65 or time 45) labor income is assumed to be zero. Table II provides the implied correlation matrix of the stochastic vector $(dz_S, dz_r, dz_\pi, dz_Q, dz_\Pi, dz_l)'$.

Given the asset price dynamics in Table I we have the following hedge portfolios. First, the portfolio hedging changes in real interest rate risk consists of a long position in the 3-year bond and a short position in the 10-year bond. This is a direct consequence of a faster mean reversion in the real interest rate than in the expected inflation rate. For example from equation (18) I see that a $C(10)/C(3) = \$2.78$ position in the 3-year bond, a $-\$1.00$ position in a 10-year bond (and $-\$0.78$ cash position) implies a zero loading on expected inflation shocks and a negative loading of $[-2.78B(3) - (-1.00)B(10)]\sigma_r = -2.13\sigma_r$ on real interest rate shocks. So when the real interest rates decreases by 1% (and consequently the investment opportunity set deteriorates), then this bond portfolio shows a positive nominal return of 2.13%. Second, the portfolio best offsetting changes in the nominal house price implied by the calibrated parameter values is a short stock, short 3-year bond, long 10-year bond (and short cash) portfolio. This can be easily obtained by solving $\sigma_F = -\theta$ in equation (18). For an investor expecting to downsize his housing position this is the appropriate hedge portfolio. For an investor who is expecting to buy a (bigger) house in the near future the opposite position is needed. However, we should bear in mind that it is just a partial hedge and that most of the housing risk is idiosyncratic and unhedgeable. Third, the financial portfolio best hedging unexpected inflation risk consist of short stocks, short 3-year bonds, long 10-year bonds. However, the part of unexpected inflation that can be hedged with financial assets is small. By assumption labor income risk cannot be hedged at all.

Table III provides the other parameter values. For the risk aversion parameter I examine two values: $\gamma = 3$ for an aggressive and $\gamma = 9$ for a more risk-averse investor. Housing preferences are $\psi = 0.2$, which is the same as in Yao and Zhang (2005) (in contrast, Cocco (2005) chooses $\psi = 0.1$). The subjective discount rate is set at $\beta = 0.96$. Following Yao and Zhang (2005), the rent rate is $\zeta = 6\%$, maintenance costs are $m = 1.5\%$, transaction

Table III. Choice of other parameters.
The table reports calibrated values for the other parameters.

Variable	Symbol	Value
Risk aversion	γ	3 or 9
Housing preferences	ψ	0.20
Subjective discount rate	β	0.96
Rental rate	ζ	6.0%
Maintenance rate	m	1.5%
Move to rent cost	ν^{rent}	1.0%
Move to own cost	ν^{own}	6.0%
Minimum down payment	δ	0.20

costs when moving to own are $\nu^{own} = 6\%$ and the down payment on the house is $\delta = 20\%$ (Cocco (2005) chooses 1%, 8% and 15% for m, ν^{own} and δ respectively). Yao and Zhang (2005) assume a zero transaction cost for moving to a rental house. Taking into account the cost of for example moving furniture and in-house painting, I consider a modest $\nu^{rent} = 1\%$ more reasonable.

4 Results

I solved the model presented in Section 2 using calibrated parameter values presented in Section 3. The solution comprises the optimal choice for tenure, house size, financial portfolio and consumption conditional on the state. The non-separable state variables for the problem are, the current tenure indicator, I, wealth to labor income ratio, y, housing to wealth ratio, h, real interest rate, r, and time. The number of state variables is too large too show the full solution in one or two graphs. Instead I illustrate the model implications in other ways. Most importantly, I simulate paths for the non-separable state variables using derived optimal choices and doing so also simultaneously obtain values for the choice variables and separable state-variables (in particular total wealth). I will show results for the mean investor, determined by averaging state and choice variables for a 1000 (simulated) investors of equal age. As starting values for the non-separable state variables at age 20 I choose $I_0 = 0$, i.e. a renter, $y_0 = 1/12$, i.e. starting wealth equal to one month salary, $h_0 = 30$, i.e. a rental house worth 30 times the starting wealth, $r_0 = \bar{r}$, i.e. the real interest rate is at his long-run average. I normalise total wealth and the house price at age 20, i.e. $w_0 = 1$ and $q_0 = 1$. In addition to the simulation exercise I will illustrate the impact of house size on optimal choices by varying the housing to wealth ratio, h, and fixing the other state variables.

Figure I: Housing and other goods consumption for the mean $\gamma = 3$ investor
Values are based on the mean of a 1000 simulations with $I_0 = 0$, $y_0 = 1/12$, $h_0 = 30$ and $r_0 = \bar{r}$ as start values at age 20. I normalise $w_0 = q_0 = 1$.

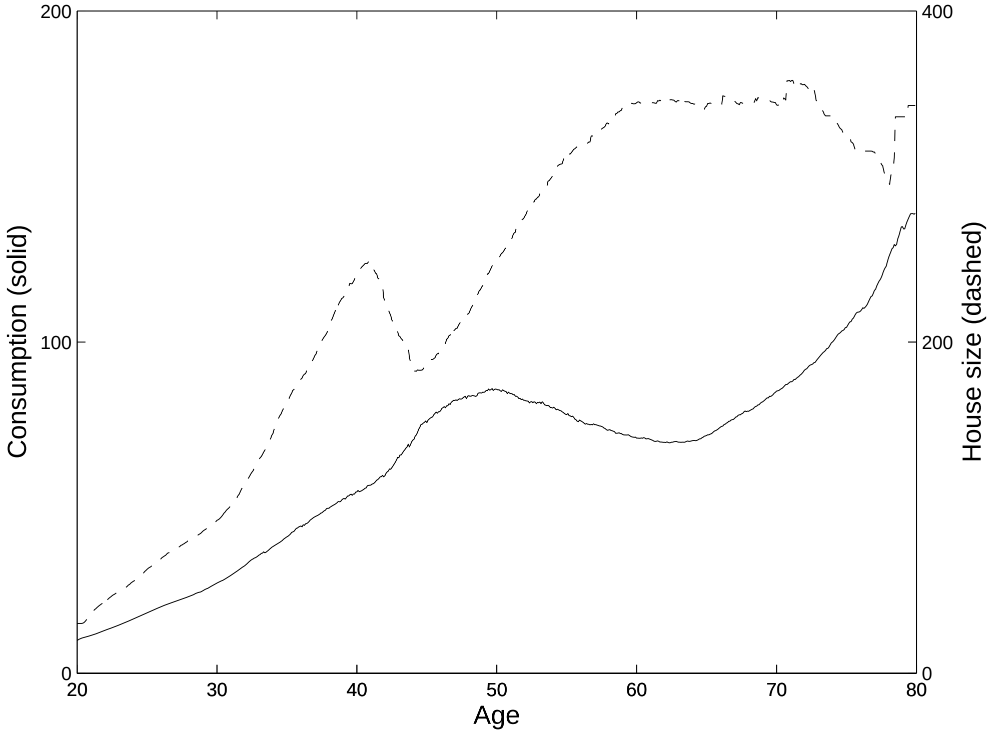


Figure II: Move rate and fraction owning for the mean $\gamma = 3$ investor
Values are based on the mean of a 1000 simulations with $I_0 = 0$, $y_0 = 1/12$, $h_0 = 30$ and $r_0 = \bar{r}$ as start values at age 20. I normalise $w_0 = q_0 = 1$. The move rate is annualised.

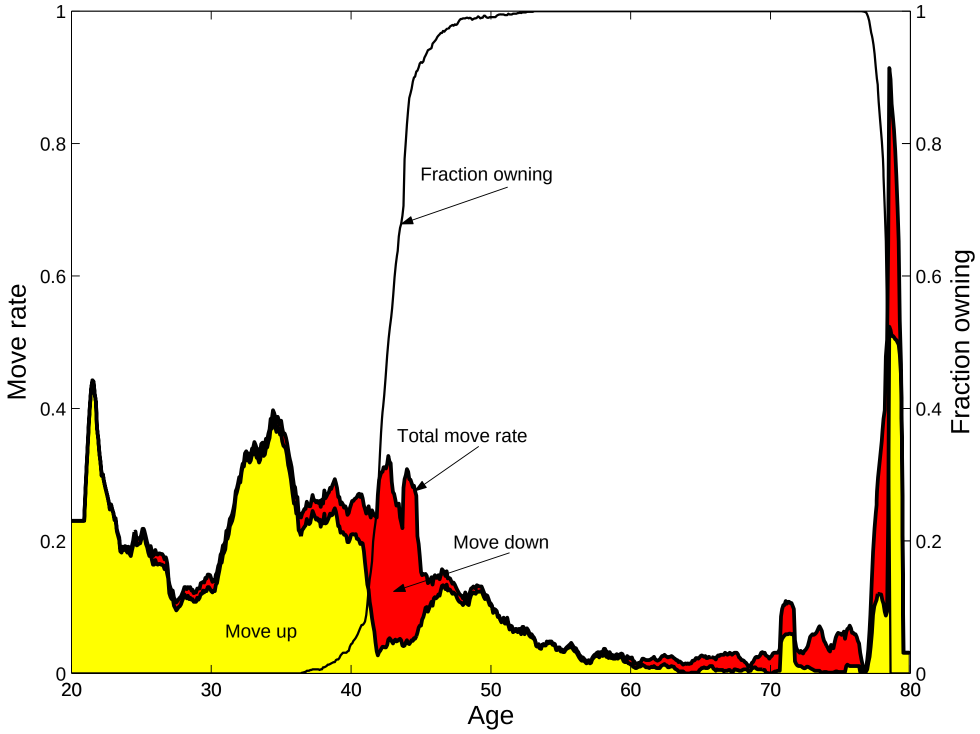


Figure I shows consumption and house size for the mean investor with risk aversion parameter $\gamma = 3$. These are the two variables that enter the investor's lifetime utility function as presented in equation (1). Figure II shows the annualised move rate for the mean investor and the fraction of investors owning the house they live in. Young investors have large human capital and little wealth, which makes them borrowing constraint. Over time the investor's wage increase and we see in Figure I both housing and other goods consumption rise between age 20 and 40. In Figure II we see that no investor owns in this phase of life. Investors have too little wealth saved to pay the down payment on a reasonable size house. Most investors move too bigger rental houses in this period though. Recall that moves are generated for endogenous reasons only in my model. Around age 40 investors buy their first home. Most of the time this is a smaller home than the one they were renting just before. We can see this by the decline in house size in Figure I or the many moves down around this age in Figure II. Owning involves lower per period costs than renting which makes investors eager to buy, even if they have not enough wealth to pay for the down payment of a house as big as the one they are renting. Around age 50 most investors own the house they live in. The mean house size rises until age 60, is then fairly constant until age 70, and then starts decreasing again. Since I do not have a bequest motive, investors want to consume all their wealth before they die. Because of the compulsory down payment on the house, the investor optimally decreases house size (and therefore down payment) and eventually starts renting towards the end of his life. In Figure II the move from own to rental house is visible by the large moving rate around age 78. Because lower per period housing than rental costs, housing wealth is released fairly late in life. This causes consumption to be large in the last period of life.

Next I'll discuss the portfolio choice and wealth accumulation for the mean investor with risk aversion parameter $\gamma = 3$ and $\gamma = 9$, presented in Figures III and IV respectively. Portfolio shares add up to one. The (negative) mortgage exactly cancels against the part of housing wealth that exceeds the dashed, horizontal line for the total portfolio share equals one. Consequently, net housing wealth is exactly the part of housing wealth that is underneath this dashed, horizontal line. I also plot total wealth.

In the first years after age 20 the investor has very little wealth. Both the aggressive $\gamma = 3$ and the more risk-averse $\gamma = 9$ investor choose a fairly conservative portfolio, considering their huge human capital. Because labor income and rent costs are risky, the holding of some wealth is partially motivated as a buffer stock. The investor cannot hedge against rental (i.e. house price) increases because this mainly involves a negative 10-year bond position. Labor

Figure III: Portfolio choice and wealth for the mean $\gamma = 3$ investor
Values are based on the mean of a 1000 simulations with $I_0 = 0$, $y_0 = 1/12$, $h_0 = 30$ and $r_0 = \bar{r}$ as start values at age 20. I normalise $w_0 = q_0 = 1$.

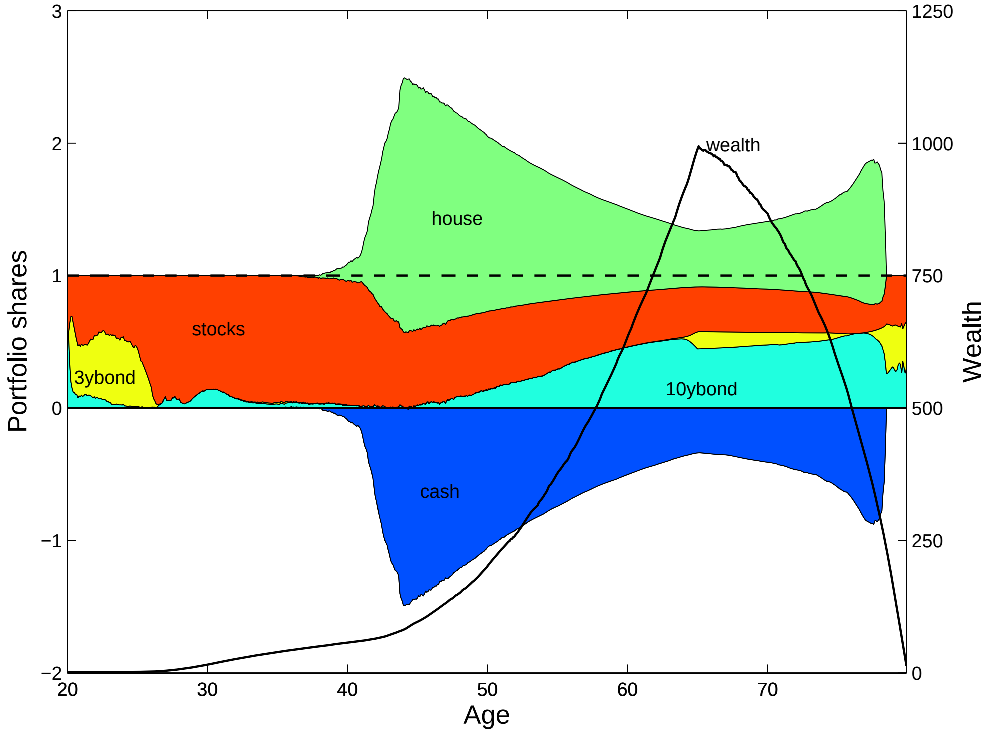
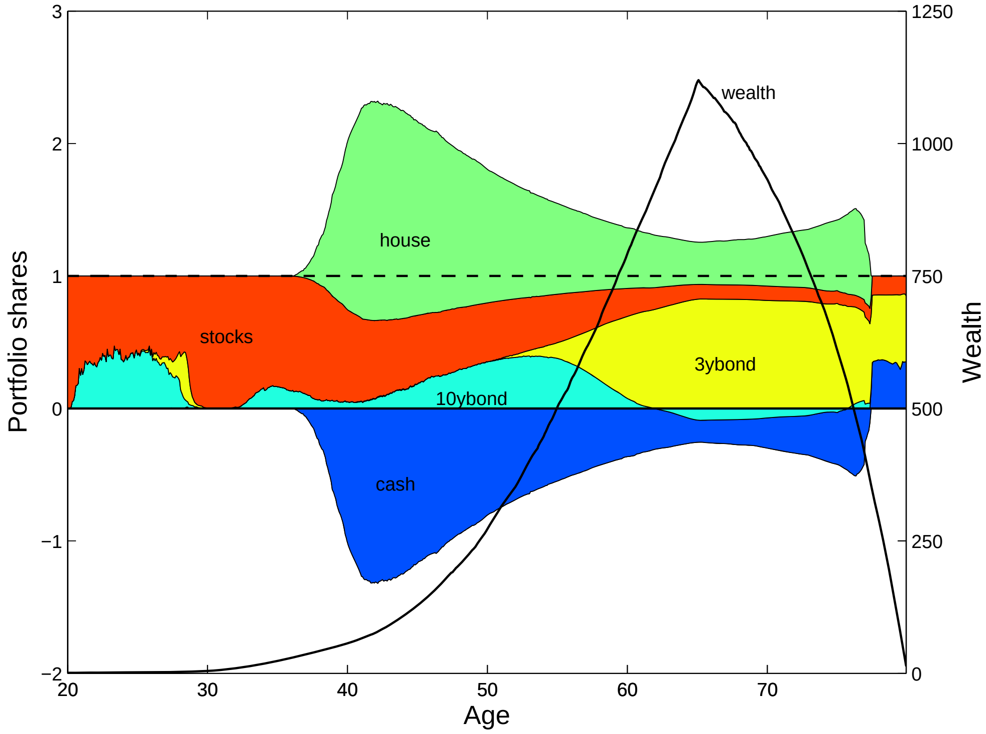


Figure IV: Portfolio choice and wealth for the mean $\gamma = 9$ investor
Values are based on the mean of a 1000 simulations with $I_0 = 0$, $y_0 = 1/12$, $h_0 = 30$ and $r_0 = \bar{r}$ as start values at age 20. I normalise $w_0 = q_0 = 1$.



income shocks cannot be hedged either. Instead the investor holds bonds to hedge real interest rate shocks and some stocks to exploit the equity premium. The aggressive $\gamma = 3$ investor mainly holds 3-year bonds for the hedge against real interest rate risk. The more risk-averse $\gamma = 9$ has a larger hedge demand and chooses 10-year bonds, that have a larger absolute loading on real interest rate risk. Between age 35 and 40 somewhat more wealth is accumulated and the buffer stock role of the wealth holding becomes less relevant. Wealth is still small compared to human capital though, creating a desire to leverage risk taking in the financial portfolio. Since stocks have the largest risk premium, most investment is in stocks in this period.

Around age 40 a house is purchased. Both the $\gamma = 3$ and the $\gamma = 9$ investor choose a pure adjustable-rate mortgage at this point in life, reflecting the desire to leverage the risk exposure. The financial portfolio still consists mainly of stocks, but there is also a small holding of 10-year bonds. The purpose of this 10-year bond holding is again to hedge real interest changes. A 3-year bond has a larger relative loading on real interest rate shocks (relative to expected inflation shocks), but 10-year bonds have a larger absolute loading on real interest rate shocks. The latter is preferred by the 40 year old investor who still has a large human capital creating a desire to leverage the stock exposure which in turn leaves little financial wealth for hedging purposes. The hedging demand is bigger for the more risk averse $\gamma = 9$ investor. As wealth is accumulated between age 40 and 65 and human capital is capitalised, the desire for leveraged stock exposure decreases and the hedge demand increases. For the $\gamma = 3$ investor this results in increasing 10-year bond holdings. The $\gamma = 9$ investor, who has less demand for stocks and therefore more financial wealth available for hedging real interest rate risk, gradually switches to 3-year bonds between age 55 and 65. In fact, the 10-year bond position becomes negative, indicating that not a pure adjustable-rate mortgage, but a hybrid mortgage is optimal here. The optimal mortgage choice at retirement is consistent with results presented in Van Hemert De Jong and Driessen (2005), who abstract from labor income. They show that there is a large welfare loss when no hybrid mortgage is available and the investor has to choose either an ARM or an FRM.

Even though the main motivation for holding bonds is the hedge against real interest changes, there is an additional effect. The investor takes into account he will downsize his housing wealth and eventually rent again during retirement. This creates a motive to hedge against falling house prices. As discussed in section 3 this calls for a long 10-year bond and a short 3-year bond position. In Figure V I present portfolio shares for a $\gamma = 3$ investor under the alternative assumption of zero correlation between the house price and financial asset

Figure V: Portfolio choice and wealth for the mean $\gamma = 3$ investor (completely idiosyncratic house risk)

Values are based on the mean of a 1000 simulations with $I_0 = 0$, $y_0 = 1/12$, $h_0 = 30$ and $r_0 = \bar{r}$ as start values at age 20. I normalise $w_0 = q_0 = 1$. I use the alternative assumption of zero correlation between the house price and asset prices, as presented in the third column of Table I.

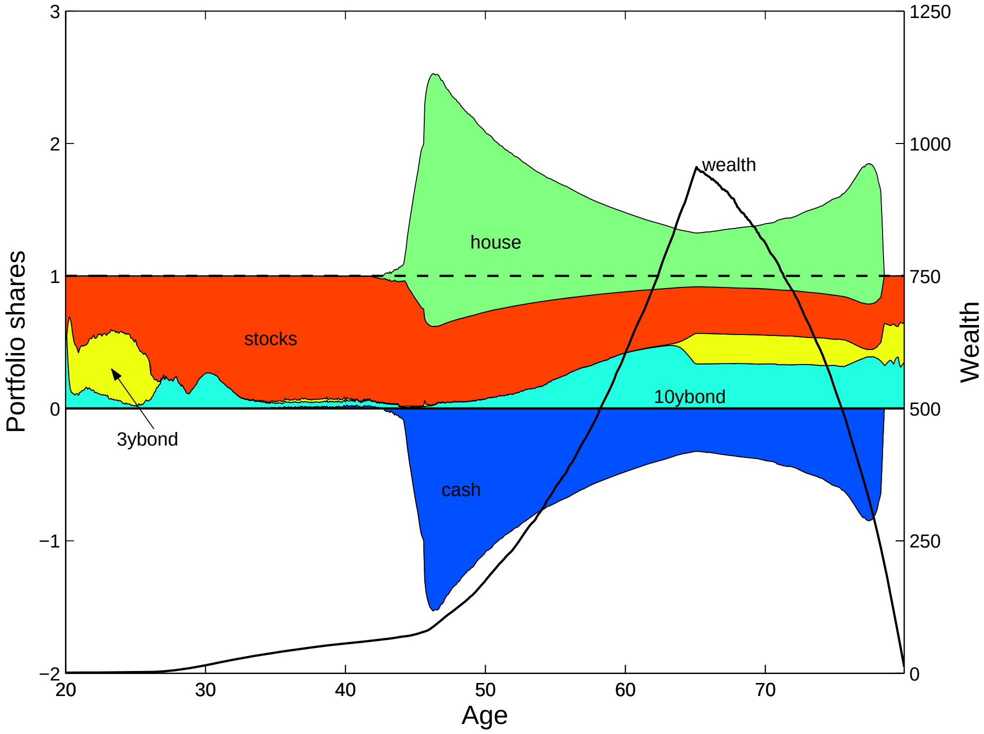


Table IV. Financial portfolio choice for different housing to wealth ratios

The table presents optimal financial portfolio choice for a homeowner of age 65 with a wealth to labor income ratio equal to the mean of the simulation analysis, an interest rate equal to the long run mean $\bar{r}$ and housing to wealth ratios ranging from 0.2 to 0.8. In addition it presents the wealth equivalent value, defined as the wealth needed to attain the same utility as with the optimal housing to wealth ratio and 100 wealth. Move indicates an investor optimally chooses to change house size.

Panel A: the investor has risk aversion $\gamma = 3$

Variable	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$	$h = 0.6$	$h = 0.7$	$h = 0.8$	$h = 0.9$
weq	move	101.55	100.49	100.00	100.12	100.67	101.40	102.04	move
$H_t^{new} q_t / \hat{w}_t$	move	0.21	0.31	0.41	0.51	0.61	0.71	0.81	move
$stock_t$	move	0.38	0.35	0.33	0.32	0.32	0.30	0.27	move
$3ybond_t$	move	0.14	0.17	0.18	0.12	0.03	0.00	0.00	move
$10ybond_t$	move	0.43	0.41	0.40	0.44	0.52	0.56	0.56	move
$cash_t$	move	−0.16	−0.24	−0.32	−0.40	−0.48	−0.56	−0.64	move

Panel B: the investor has risk aversion $\gamma = 9$

Variable	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$	$h = 0.6$	$h = 0.7$	$h = 0.8$	$h = 0.9$
weq	move	100.45	100.00	100.44	101.26	move	move	move	move
$H_t^{new} q_t / \hat{w}_t$	move	0.21	0.31	0.41	0.51	move	move	move	move
$stock_t$	move	0.12	0.11	0.10	0.11	move	move	move	move
$3ybond_t$	move	0.84	0.83	0.81	0.79	move	move	move	move
$10ybond_t$	move	−0.12	−0.10	−0.07	−0.01	move	move	move	move
$cash_t$	move	−0.04	−0.14	−0.25	−0.39	move	move	move	move

prices. Indeed the 10-year bond allocation is lower and the 3-year bond allocation larger without this additional hedge demand. This hedge demand is also detectable in Figures III and IV, where the 10-year bond position decreases and the 3-year bond position increases once the house is sold. Further comparing Figures III and V we see little differences between the two graphs earlier in life, indicating that hedging house price risk with financial assets does not play a very important role in the accumulation phase of life.[7]

Having illustrated the model implications for the mean investor using a simulation analysis, I now turn my attention to the impact of house size on optimal portfolio choice. Table IV shows the optimal portfolio choice for a homeowner at retirement age (65) for different housing to wealth ratios. Again I consider both an aggressive $\gamma = 3$ (Panel A) and a more risk-averse $\gamma = 9$ (Panel B) investor. The wealth to labor income ratio and the real interest rate are fixed and set at the mean value of the previous simulation exercise. That is, $y = 12$ and $r = \bar{r}$ for both the $\gamma = 3$ and $\gamma = 9$ investor. Notice that the housing allocation $H_t^{new} q_t / \hat{w}_t$ is slightly above the housing wealth ratio because the latter is measured relative to start of period wealth (w_t) while the first is measured relative to wealth after consumption and housing costs ($\hat{w}_t$). I also report the wealth equivalent value defined as the wealth needed to attain the same utility as with the optimal housing to wealth ratio and 100 wealth. I have

$$weq\left(h\right) = 100 * \left[\tilde{J}\left(0, 12, h^{optimal}, \bar{r}, 45\right) / J\left(0, 12, h, \bar{r}, 45\right)\right]^{1/(1-\gamma)}, \qquad (39)$$

where I use $h^{optimal}$ for the housing to wealth ratio that is optimal given the values of the other state variables.

At any time an investor has the possibility to move to the house size that is optimal given the values of the other state variables. However, because moving involves transaction costs, there is range for the housing to wealth ratio where the investor optimally chooses not to move. For the $\gamma = 3$ investor this is $h \in [0.2, 0.8]$. When the housing to wealth ratio is outside this range, he will optimally choose to move to a house that brings him inside the range again. The range is more narrow for the more risk averse $\gamma = 9$ investor compared to the aggressive $\gamma = 3$ investor. The optimal housing to wealth ratio is lower for the $\gamma = 9$ investor. At this age the investor is over exposed to house price risk, considering the limited (house price dependent) housing costs during residual life. This makes a more risk-averse investor less willing to own a large house.

[7] As Sinai and Souleles (2004) notice, owning itself may provide a hedge against future housing costs risk, which in turn might influence the tenure decision.

For larger housing to wealth ratios the investor can take a larger mortgage. However, because of the required down payment, the investor has less wealth available to take long positions in financial assets. Recall that the size of the mortgage may not exceed $1-\delta$ times the housing wealth at any time, not only at moments the investor adjusts his mortgage size. With less financial wealth available the investor tends to shift his bond portfolio to long-term bonds which have a larger absolute loading on real interest rate risk. Doing so he can maintain the appropriate hedge against changing interest rates. This results in the general tendency to increase the maturity of bond portfolio with the housing to wealth ratio. Stock allocation tends to be crowded out more for larger housing to wealth ratios. However, there is an additional, superposed, effect that blurs the picture somewhat. Investors act less risk averse close to the border of the no-move region than close to the optimal housing to wealth ratio.[8] In Panel A this is best visible by the high 10-year bond allocation at the left border of the no-move region ($h = 0.2$). In Panels B it is more clearly visible by the slightly rising stock allocation near the right border of the no-move region ($h = 0.4$ vs. $h = 0.5$).

5 Conclusion

I investigated housing and portfolio choice under stochastic inflation and real interest rates. Both housing and financial portfolio choice show a clear life-cycle pattern. When just entering the labor force an investor is very borrowing constrained and prefers to rent. After having saved for the down payment he buys a house and enjoys lower per period, per unit housing cost. At the very end of life he starts renting again, which enables him to consume the down payment on the previously owned house.

Young homeowners choose an ARM and invest financial wealth mainly in stocks. At retirement bonds play an important role, mainly as hedge against changes in the real interest rate. The mean-reversion in the real interest rate is faster than in the expected inflation rate. This implies that the sensitivity to real interest rate shocks relative to the sensitivity to expected inflation rate shocks will be higher for short-term bonds than for long-term bonds. The absolute sensitivity to both shocks is higher for a long-term bond though. An aggressive investor, who is still very financially constrained at retirement, mainly chooses 10-year bonds for the hedge against real interest rates and continues to finance his house with an ARM. A more risk-averse investor prefers short-term bonds to hedge real interest

[8] See e.g. Grossman and Laroque (1990) for a study on optimal behavior in the presence of an illliquid asset like a house.

changes and switches to a hybrid mortgage, being a combination of an ARM and an FRM.

The choice on mortgage type is first and foremost a choice between different interest rate products and, as I showed, should therefore be analysed in conjunction with the other financial decisions. However, there might be additional effects from which I abstracted in the current analysis. First, the payments on an FRM are higher than on an ARM for a normal, upward-sloping, nominal interest rate curve. In countries where mortgage payments are tax deductible this might result in larger tax benefits for homeowners financing their house with an FRM. Second, holders of an FRM might have a prepayment option, which in turn will give rise to a premium on the mortgage payments. Third, in reality some homeowners default on their mortgage. Incoporating these effects is a challenging avenue for future research.

6 References

Bernheim, Douglas D. (1991), How strong are bequest motives? Evidence based on estimates of the demand for life insurance and annuities, *Journal of Political Economy*, 99(5), 899-927.

Bodie, Zvi, Robert C. Merton, William F. Samuelson (1992), Labor supply flexibility and portfolio choice in a life cycle model, *Journal of Economic Dynamics and Control*, 16, 427-449.

Brennan, Michael J., and Yihong Xia (2002), Dynamic asset allocation under inflation, *Journal of Finance*, 57 (3), 1201-1238.

Brueckner, Jan K. (1997), Consumption and investment motives and the portfolio choices of homeowners, *Journal of Real Estate Finance and Economics*, 15(2), 159-180.

Campbell, John Y., and João F. Cocco (2003), Household risk management and optimal mortgage choice, *Quarterly Journal of Economics*, 118(4), 1449-1494.

Campbell, John Y., and Luis M. Viceira (2002), *Strategic asset allocation: portfolio choice for long-term investors*, New York: Oxford University press.

Campbell, John Y., and Luis M. Viceira (2001), Who should buy long-term bonds?, *American Economic Review*, 91(1), 99-127.

Cocco, João F. (2005), Portfolio choice in the presence of housing, *Forthcoming in the Review of Financial Studies.*

Cocco, João F., Francisco J. Gomes, Pascal J. Maenhout (2005), Consumption and portfolio choice over the life-cycle, *Forthcoming in the Review of Financial Studies.*

Flavin, Marjorie, and Takashi Yamashita (2002), Owner-occupied housing and the composition of the household portfolio, *American Economic Review*, 92, 345-362.

Grossman, Stanford J., and Guy Laroque (1990), Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods, *Econometrica*, 58 (1), 25-51.

Hu, Xiaoqing (2003), Portfolio choices for homeowners, Working paper.

Hurd, Michael D. (1989), Mortality risk and bequests, *Econometrica*, 57(4),779-813.

Munk, Claus and Carsten Sørensen (2005), Dynamic asset allocation with stochastic income and interest rates, Working Paper.

Nelder, J.A. and R. Mead (1965), A simplex method for function minimization, *Computer Journal*, 7, 308-313.

Sinai, Todd, and Nicholas S. Souleles (2004), Onwer-occupied housing as a hedge against rent risk, Working Paper Federal Reserve Bank of Philadelphia.

Van Hemert, De Jong and Driessen (2005), Dynamic portfolio and mortgage choice for homeowners, Working Paper.

Viceira, Luis M. (2001), Optimal portfolio choice for long-horizon investors with nontradable labor income, *Journal of Finance*, 56 (2), 433-470.

Yao, Rui, and Harold H. Zhang (2005), Optimal consumption and portfolio choices with risky housing and borrowing constraint, *Review of Financial Studies*, 18(1), 197-239.