Optimal Investment Strategies for Product-Flexible and Dedicated Production Systems Under Demand Uncertainty*

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Abstract

This paper studies the optimal investment strategy of a firm with the managerial freedom to acquire either flexible or dedicated production capacity. Flexible capacity is more expensive but allows the firm to switch costlessly between products and to handle changes in relative volumes among products in a given product mix. Dedicated capacity restricts the firm to the manufacture of a specific product but has a lower acquisition cost. Specifically, I model the investment decision of a monopolist selling two products in a market characterized by price-dependent and uncertain demand, in a continuous time setting.

I find that flexibility pays off especially when uncertainty is high, substitutability is low, and the profit levels of the two products are substantially different. In the flexible case, the firm simply produces the more profitable product under high demand, while if demand is low it produces both products to increase the total market demand. In the dedicated case the firm invests in the manufacture of both products only if the substitutability rate is low and the profitability of both products sufficiently high. Otherwise, it restricts investment to the more profitable product.

Considering a firm's decision to change from dedicated to flexible capacity, I show that despite perfectly positively correlated demand a firm will switch from dedicated to flexible capacity even for very low demand if the profitability of the products is substantially different. The option to increase the total capacity accelerates investment in flexible capacity when the profit levels of both products are sufficiently high.

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1 Introduction

Flexibility in manufacturing operations is becoming increasingly important to industrial firms. The increasing demand volatility, the internationalization of markets and competition, and shorter product life cycles pose new challenges for companies. Since the investment cost of flexible capacity usually exceeds the investment cost of dedicated capacity, firms need to know the value of flexibility. This paper focuses on the effect of demand volatility and product combination on a firm's investment decisions and the value of product flexibility. Product flexibility is considered one of the most (if not the most) strategically important types of flexibility (see for example Goyal and Netessine (2007), Jordan and Graves (1995)).

The automotive industry is a good example of an industry where manufacturers' decisions on investment in production capacity and on the optimal level of flexibility are critical. On the one hand, expanding already installed capacity is expensive (Andreou (1990)), and therefore the installed capacity must be sufficient for the entire life cycle of the product and easily adaptable to new product lines. On the other hand, the profitability of the products is threatened by low utilization of capacity as well as under-capacity. Japanese carmakers very early implemented the concept of flexibility, which gave them a significant advantage over their US and European competitors that traditionally built plants dedicated to producing a single car model. Later, the European and American car industry started to increase flexibility. The BMW group, for example, recently advertised that a new factory in Leipzig added new production capacity with a high level of flexibility[1]. However, Japanese companies still lead in this area, which is according to Goyal, Netessine, and Randall (2006) "an advantage that is at least partially responsible for the increasing market share of the Japanese carmakers."

The investment in and management of flexible capacity has received significant attention in the operations management literature. While early research in this field focused on scenarios with exogenously given prices and static time models, recent papers investigate responsive pricing and multi-stage decision problems. The structure of these multi-stage decision models is: first invest in capacity, then receive additional information, and finally exploit the capacity optimally according to the revealed information. While this structure reflects that of real option models, the models are restricted to a single period and do not take timing into account. This paper applies a continuous time setting as is done in the real option literature, which allows insight into the optimal timing of a firm's investment decision. The theory of real options explores flexibility mainly in relation to the timing of capacity acquisition and does not deal with technologies that exhibit flexibility per se. This paper presents a model that takes into account both aspects: flexibility in timing and investment

[1] The BMW group states that the production program includes both the full range of cars of the BMW 1 Series (three-door, coupé, and convertible) and the BMW X1. At the Volkswagen plant in Zwickau (Germany) Passat and Golf run off the same assembly line, and the supply to the production line is also fully flexible. This allows changing the model or the interior equipment without any adaption costs or time lags. See http://www.bmwgroup.com/d/nav/index.html

in flexible capacity.

Specifically, this paper studies the investment decision of a monopolist with the managerial freedom to acquire either flexible or dedicated manufacturing capacity, in a continuous time setting. Flexible capacity allows the firm to manufacture all of its products with the same production facility while dedicated capacity restricts the firm to one product. Product flexibility has not been clearly defined in the literature. For the purpose of this study, it is defined to be a system's ability to switch costlessly between products and to handle changes in relative volumes among products. Products differ in substitutability and profitability in the market. The firm wants to protect efficiently against uncertainty in demand for all of its products. It can choose the timing as well as the quantity of the investment and is free to invest in flexible or dedicated production capacity, with the goal being to maximize expected profit. In a further step I change the initial conditions, assuming that the firm has entered the market at an earlier stage and is currently producing with dedicated capacity. The firm now has the option of investing in flexible capacity.

I analyze the results with a focus on four possible product combinations. The specific combination will turn out to be a key factor in the firm's investment decision. The four cases have different profitabilities and substitutability rates between the products: (1) The first is a combination of two similarly profitable products with low substitutability. Volkswagen's models Passat and Golf are an example of such a combination. In the Volkswagen plant in Mosel these two models are produced on an assembly line that can switch between the two models without time lags or costs. (2) National-brand manufacturers that produce both brand products and private-label products are an example of firms that produce two highly substitutable products for the same market with different profitabilities. Their own brand is more profitable for them than the product that is marketed by a private-label retailer. For example, Danone, the famous French corporation, produces a version of its popular cream-cheese dessert "Fruchtzwerge" for the private label "Desira" of Germany's biggest discounter Aldi[2]. (3) The third case is a combination of two products for the same market that are almost equally profitable and highly substitutable. For brand manufacturers with less successful national brands, so-called B and C brands, the production of private labels can be almost as profitable as producing their own brand. For example, Concorp group, a Dutch confectionery company, states publicly that they produce for dual branding[3]. (4) The fourth case is a combination of two products with low substitutability and a substantial difference in profitability. For example, a technology company may produce an up-to-date touchscreen mobile phone and a less successful older model in the same facility.

[2]See p. 39 in Schneider, Martina, "Aldi - Welche Marke Steckt Dahinter? 100 Aldi-Top-Artikel und Ihre Prominenten Hersteller," Muenchen: Suedwest Verlag.

[3]The website of the Concorp group states that they "...build brands and deliver private label concepts with added value in all segments of selected national and international confectionery markets." Concorp group produces candy foam and boiled sweets under three different brand names in its production site in Waddinxveen. See http://www.concorp.nl/international/pdf/concorp_international_overview.pdf

This paper focuses on the effect of demand variability—a key driver of flexibility—together with substitutability and product-profitability effects. I show that in the flexible case, under high demand the firm simply produces the more profitable product; if demand is low the firm produces both products to increase the total demand. If we compare the optimal flexible investment strategies for the above four cases, we see that a firm selling two products with low substitutability and a large profitability difference invests in significantly more capacity. Capacity grows more than proportionally with the uncertainty level. This confirms the intuition that a firm producing two almost equally profitable products with low substitutability profits the most from the potential to increase the market size by producing both products. In the dedicated case, the firm invests in both capacities if the substitutability is low and the profitability of both products sufficiently high. Otherwise, it ignores the demand for one product and installs dedicated capacity for the more profitable product. In this case, the firm can gain from the downside potential only when demand is very low and therefore the negative effect of the requirement to produce at full capacity dominates. For both dedicated and flexible capacity investment, I show that the firm invests later in capacity if demand uncertainty increases. This result is also obtained for production-flexible capacity investment by Hagspiel, Huisman, and Kort (2011) and Dangl (1999).

To study the value of flexibility, I consider as a benchmark the situation in which a firm relies on at most two dedicated capacities rather than on one flexible capacity. Flexibility pays off especially when uncertainty is high, substitutability is low, and the profit levels of the two products are substantially different. The flexible firm can increase its total market demand if demand falls by producing the less profitable second product. The dedicated firm can not, because its capacity is dedicated to the more profitable product.

Firms often do not face a direct choice between dedicated and flexible capacity but must evaluate whether or not to invest in flexible capacity while currently carrying out dedicated production. I therefore also analyze this scenario. This study shows that despite the assumption of perfectly positively correlated product demand, a firm will switch to flexible capacity even under low demand if the profitability of the products is substantially different. I conclude that the specific product combination has a crucial effect on the investment decision. Therefore, when choosing between dedicated and flexible capacity firms should take into account not only the demand volatility and correlation but also the specific product combination. This contradicts early literature claiming that flexibility has no value in the case of perfectly positively correlated demand and confirms more recent literature that claims that flexibility does have value in this situation. I extend this approach by analyzing the effect of specific product combinations on the value of flexibility.

As mentioned earlier, there are two streams of literature that are relevant to this study: the first considers the issue of product flexibility from an operations management perspective, while the second studies capital budgeting decisions under real option theory. Both will now be discussed.

The issue of resource flexibility attracted interest from the operations management community at the beginning of the 1990s, following the increasing viability of flexible, computer-controlled manufacturing systems. The work in this area was initiated by Fine and Freund (1990). They use a two-stage convex quadratic program to derive necessary and sufficient conditions for the acquisition of flexible capacity. In the first stage a technology investment decision is made. The demand realization is then observed, and an optimal production decision is made at the second stage. Inspired by Fine and Freund, Van Mieghem and Dada (1999) present closely related work that disproves Fine and Freund's claim that flexible capacity does not provide additional value when the product demands are perfectly positively correlated. They show that in addition to its adaptability to demand-mix changes, product-flexible technology provides an opportunity for revenue improvement through its ability to exploit differentials in the price (margin) mix. They argue that product flexibility introduces the option of producing and selling more highly profitable products at the expense of less profitable products. They show that this option can be valuable even for perfectly positively correlated product demand. I extend this claim by analyzing specific differences in product combinations and their individual effects on the value of flexibility.

Most closely related to my work are the papers of Chod and Rudi (2005) and Bish and Wang (2004), who study the resource investment decision of a two-product, price-setting firm that operates in a monopolistic setting. Chod and Rudi look at the effect of demand variability and demand correlation on the optimal flexible-resource investment decision and show that the expected profit increases with variability and decreases with the correlation of the normally distributed demand. Bish and Wang's model is more general: it allows the firm to invest in flexible and dedicated resources at the same time but does not include cross-price effects. Both papers present two-stage models that provide insight into the optimal resource size and allocation but do not consider the timing aspect. In contrast, I focus on an economic environment where uncertainty in the demand for two products arises from a single market. Optimizing investment decisions under uncertainty about the general economic situation is even more appealing since industry has recently witnessed one of the largest economic crises in history. The credit crunch affected the entire global market, and companies faced large drops in demand for their products.

Applying a continuous time setting provides insight into the optimal timing of the firm's investment strategy. The theory of real options is used to evaluate investment decisions with the following three characteristics: (1) the investment is irreversible, (2) there is uncertainty about future rewards, and (3) the timing of the investment is flexible. However, real option theory explores flexibility mainly in relation to the timing structure of capacity or information acquisition or resource commitment, i.e., the firm loses flexibility when it makes an irreversible commitment. Most papers do not deal with technologies that exhibit flexibility per se. Now that more and more firms invest in flexible capacity because it appears to be necessary to hedge against highly volatile demand, it is important to develop existing models further with special attention to flexible capacity. This paper aims to take a crucial step in this direction: I explicitly consider the use of a (product-) flexible technology.

Real option papers that consider investments in flexible capacity usually take into account capacity that allows switching between different inputs or different outputs. With regard to product flexibility this means that the system is able to switch from producing product 1 to producing product 2 and vice versa. In contrast, I consider a system that is flexible in adapting the relative volume of two products, i.e., it can use the available capacity to produce x units of one product and y units of the other and can continuously adapt these quantities over time. Early approaches to the evaluation of such switching options were presented by McDonald and Siegel (1986), Kulatilaka (1988), and Triantis and Hodder (1990). Conceptually, the switch between two volatile assets or commodities can be modeled as an exchange option. Margrabe (1978) and McDonald and Siegel (1986) model European finite and American perpetual exchange options, respectively, which are linearly homogeneous in the underlying stochastic variables.

Triantis and Hodder (1990) evaluate product(-mix) flexibility based on option principles. Kulatilaka (1988) applies option-pricing principles using a stochastic dynamic programming formulation that includes costly switching between modes of operation. Andreou (1990) published a more applied study associated with General Motors Research Laboratories focusing on the economic evaluation of product flexibility. He presents a financial model for calculating the dollar value of flexible plant capacity for two products under conditions of uncertain market demand. In recent work Dockendorf and Paxson (2011) present a two-factor model with continuous switching opportunities between two commodity outputs, taking into account operating and switching costs and the possibility of suspension of operation. Adkins and Paxson (2011a) evaluate input-switching options for single and multiple switching. Both papers present quasi-analytical solutions for two-factor models. Other recent papers that deal with options for switching between inputs, between outputs, or between inputs and outputs include Adkins and Paxson (2011b), Sigbjorn, Steen, and Roar (2008), and Bastian-Pinto, Brandio, and Hahn (2009). These papers evaluate investment in flexible technology based on option theory, but do not consider the timing decision or the capacity choice.

This work was inspired by the increasing interest shown by the operations and production management sector in the development of real option theory for technological flexibility. Bengtsson (2001) relates the real option literature to manufacturing flexibility from an industrial engineering/production management perspective. He lists product flexibility as one of the flexibility types that had not at that time been treated as real options. While his work addresses a wide range of manufacturing flexibility, Bengtsson and Olhager (2002) use real option theory to evaluate a specific type, i.e., product-mix flexibilities, in a real case analysis. Their main focus is on finding the value of a production system with multiple products that is applied to real case data; the timing and capacity decisions are not considered. There is an increasing number of real cases and empirical analysis in this area. Two recent papers are Goyal, Netessine, and Randall (2006) and Fleischmann, Ferber, and Henrich (2006). Both focus on the automotive industry.

This paper is organized as follows. The next section presents the general model and solves the optimization problems for both flexible capacity and dedicated capacity. The optimal investment triggers for size and time of investment are derived. Section 3.1 analyzes the capacity and timing decisions for flexible capacity investment and shows how these decisions are affected by demand uncertainty. Section 3.2 concentrates on dedicated production capacity. Section 4 studies the optimal investment strategy of a firm deciding between flexible and dedicated capacity investment and quantifies the value of flexibility. Section 5 studies the optimal investment in flexible capacity assuming that the firm is currently using dedicated capacity. Section 6 provides concluding remarks.

2 Model

Consider a firm that produces two products, A and B. The firm wishes to make an optimal capacity investment. This involves three decisions: when to invest, the capacity, and in which type of capacity to invest. The firm can invest in at most two dedicated capacities, each of which can produce only one product, or in a more expensive, flexible production capacity, which can produce both products.

The firm is uncertain about future demand; the inverse demand functions are assumed to be linear. The inverse demand functions for the two products are given by

$$p_A(\theta_t, q_A, q_B) = \theta_t - q_A - \gamma q_B, \tag{1}$$

$$p_B(\theta_t, q_B, q_A) = \alpha\theta_t - q_B - \gamma q_A, \tag{2}$$

where the demand intercept θ follows geometric Brownian motion:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t. \tag{3}$$

In this expression μ is a constant representing the trend, σ is the uncertainty, and dW_t is the increment of a Wiener process implying that it is independently and normally distributed with mean 0 and variance dt. I will often refer to the uncertainty in the demand intercepts simply as the "demand uncertainty." $\gamma \in (-1, 1)$ is the product substitutability, and $\gamma > 0$ ($\gamma < 0$) signifies that the products are substitutes (complements). Since products made by the same flexible resource tend not to be complements, most applications are characterized by a nonnegative γ. Therefore, this work focuses on the case where the products are substitutes. The two products are assumed to be sold in the same market. Product A is the more profitable product in this market, i.e., $\alpha < 1$. $\alpha \in (0, 1)$ is referred to as the profitability of product B. Let the production quantity of product A (B) at time t be $q_{t,A}$ ($q_{t,B}$); I will drop the time subscript whenever there can be no misunderstanding. For simplicity, variable production costs are not considered. It follows that the profit flow is defined by

$$\Pi(\theta) = \max_{q_A, q_B} [p_A q_A + p_B q_B]. \tag{4}$$

The total production output, i.e., $q = q_A + q_B$, is restricted to be the full capacity. This means that after investment the firm always produces at full capacity, a constraint also referred to as the capacity clearance. Two considerations motivate the introduction of this assumption here. First, it allows a focus on the effect of a specific type of technological flexibility, i.e., product-flexibility, in the analysis. Removing this assumption and therefore allowing the firm to produce below capacity would add production flexibility to the problem. See Anupindi and Jiang (2008) and Hagspiel, Huisman, and Kort (2011) for discussions of production-flexible capacity. Second, producing below capacity often has large fixed costs associated with, for example, labor and production ramp-up. Therefore, in practice firms often reduce prices to keep production lines running. See Goyal and Netessine (2007) and Chod and Rudi (2005) for examples of similar assumptions.

The flexible capacity is denoted by K_F and the dedicated capacities by K_{D_A} and K_{D_B}, respectively. The investment costs are sunk and assumed to be linear (for this assumption see, for example, Fine and Freund (1990), Van Mieghem and Dada (1999), and Chod and Rudi (2005)). Let c_i be the unit cost of investing in resource K_i, $i = F, D_A, D_B$.

2.1 Flexible Capacity

Consider a firm that must decide about investment in flexible capacity. Flexible capacity allows it to produce both products, A and B, on the same production line. It must choose the optimal time to invest and the optimal capacity, considering that it must produce at full capacity after the moment of investment. The optimal output rate for the two products, q_A^* and q_B^*, is determined by maximizing the profit flow given the capacity clearance constraint ($q_A + q_B = K_F$) and the upper and lower bounds, $0 \leq q_B, q_A \leq K_F$. This gives

$$q_A^* = \begin{cases} \frac{\theta(1-\alpha)}{4(1-\gamma)} + \frac{K_F}{2} & \text{for } \theta < \hat{\theta}, \\ K_F & \text{for } \theta \geq \hat{\theta}, \end{cases} \tag{5}$$

$$q_B^* = \begin{cases} -\frac{\theta(1-\alpha)}{4(1-\gamma)} + \frac{K_F}{2} & \text{for } \theta < \hat{\theta}, \\ 0 & \text{for } \theta \geq \hat{\theta}, \end{cases} \tag{6}$$

where $\hat{\theta}$ is $\frac{2(1-\gamma)}{(1-\alpha)} K_F$. For low demand, $\theta \in [0, \hat{\theta})$, the firm will produce both products. If the demand increases, i.e., $\theta \in [\hat{\theta}, \infty)$, the firm will use the full capacity K_F for production of the more profitable A and suspend production of B. Expressions (5) and (6) imply that the profit flow is given by

$$\Pi(\theta) = \begin{cases} \frac{(1-\alpha)^2}{8(1-\gamma)}\theta^2 + \frac{(1+\alpha)}{2}\theta K_F - \frac{(1+\gamma)}{2}K_F^2 & \text{for } \theta < \hat{\theta}, \\ (\theta - K_F) K_F & \text{for } \theta \geq \hat{\theta}. \end{cases} \tag{7}$$

To find the value of this investment project, dynamic programming is applied. The value function, denoted here by $V(\theta, K_F)$, must satisfy the Bellman equation

$$V(\theta, K) = \pi(\theta, K)dt + E\left[V(\theta + d\theta, K)e^{-rdt}\right], \tag{8}$$

where r is the (constant) discount rate. Applying Ito's Lemma and substituting and rewriting leads to the differential equation (see, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \mu\theta\frac{\partial V}{\partial\theta} - rV + \Pi(\theta) = 0. \tag{9}$$

Solving this equation for $V(\theta, K)$, given that we have two different regions, and ruling out bubble solutions, we get the following value for the project:

$$V(\theta, K_F) = \begin{cases} A_1(K_F)\theta^{\beta_1} + a_1\theta^2 + a_2\theta K_F + a_3 K_F^2 & \text{for } \theta < \hat{\theta}, \\ B_2(K_F)\theta^{\beta_2} + \frac{\theta K_F}{r-\mu} - \frac{K_F^2}{r} & \text{for } \theta \geq \hat{\theta}, \end{cases} \tag{10}$$

with $a_1 = \frac{(1-\alpha)^2}{8(1-\gamma)(r-2\mu-\sigma^2)}$, $a_2 = \frac{(1+\alpha)}{2(r-\mu)}$, and $a_3 = -\frac{(1+\gamma)}{2r}$. β_1 (β_2) is the positive (negative) root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0. \tag{11}$$

$V(\theta)$ must be continuously differentiable across the boundary $\hat{\theta} = \frac{2(1-\gamma)}{(1-\alpha)}K_F$. Using this fact we can derive the constants A_1 and B_2:

$$\begin{aligned} A_1(K_F) &= K_F^{2-\beta_1}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)}\right. \\ &\quad \left. -\frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right], \end{aligned} \tag{12}$$

$$\begin{aligned} B_2(K_F) &= K_F^{2-\beta_2}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)}\right. \\ &\quad \left. -\frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right]. \end{aligned} \tag{13}$$

Corollary 1 in Appendix A shows that A_1 is negative and B_2 positive for all parameter values.

The value of the investment project in the region $\theta < \hat{\theta}$ consists of four terms where the last three terms constitute the cash flow generated by the sales. The first term $A_1(K_F)\theta^{\beta_1}$, which is negative, corrects for the fact that in a mathematically optimal case the production of product B would become negative for $\theta > \hat{\theta}$. Economically this does not make sense and therefore the output quantity is constrained by $q_B^* \geq 0$. The absolute value of this term decreases with θ.

In the region $\theta \geq \hat{\theta}$, demand is high and the firm uses all of its installed capacity to produce the more profitable A. This generates a discounted cash flow stream that is reflected in the second and third terms of the value of the investment project associated with this region. The first term, $B_2(K_F)\theta^{\beta_2}$, describes the option value that accounts for the possibility that if demand decreases the company can again produce both products and therefore gain revenue. This option value is decreasing for large θ.

Given the value of the project, $V(\theta, K_F)$, we can derive the optimal investment strategy. In general the procedure is as follows. First, the optimal capacity $K_F^*(\theta)$ is determined for a given level of θ by setting the marginal value of the project equal to the marginal investment costs c_F. Second, we derive the optimal investment threshold θ^*. For this demand level θ^* the firm is indifferent between investment and waiting to invest. Investment (waiting) is optimal for a θ larger (smaller) than θ^*.

Investment can take place either in region I, i.e., $\theta < \hat{\theta}$, or in region II, i.e., $\theta \geq \hat{\theta}$. For $\theta < \hat{\theta}$ the firm uses the newly acquired capacity to produce both products immediately after making the investment. For $\theta \geq \hat{\theta}$ the entire capacity is used to produce A.

The following proposition provides equations that implicitly determine the threshold θ_F^* and the corresponding capacity $K_F^*(\theta^*)$ in each of the two cases. The optimal investment decision corresponds to the case that provides the larger expected value of the investment project.

Proposition 1 *The firm has two possible investment policies:*

1. *If the firm produces a positive amount of each product immediately after the investment, the optimal capacity $K_F^*(\theta)$ is implicitly determined by*

$$\frac{\partial A_1}{\partial K_F}\theta^{\beta_1} + a_2\theta + 2a_3K_F^* - c_F = 0. \tag{14}$$

If K^* is not an interior solution of the considered region, i.e., if $K^* > \frac{(1-\alpha)\theta}{2(1-\gamma)}$, the optimal capacity is replaced by the boundary solution, i.e., $\theta\frac{(1-\alpha)}{2(1-\gamma)}$. Thus, the optimal capacity for the demand realization θ is given by

$$K^*(\theta) = \max\left[K^*, \theta\frac{(1-\alpha)}{2(1-\gamma)}\right]. \tag{15}$$

The investment threshold θ^* is implicitly determined by

$$\begin{aligned} a_1(\beta_1 - 2)\theta^{*2} \quad &+ \quad a_2(\beta_1 - 1)\theta^* K_F^*(\theta^*) + \beta_1 a_3 K_F^*(\theta^*)^2 \\ &- \quad \beta_1 c_F K_F^*(\theta^*) = 0. \end{aligned} \tag{16}$$

2. *If the firm uses the entire capacity to produce the more profitable A immediately after the investment, the optimal capacity $K_F^*(\theta)$ is implicitly determined by*

$$\frac{\partial B_2}{\partial K_F}\theta^{\beta_2} + \frac{\theta}{r-\mu} - \frac{2}{r}K_F^* - c_F = 0. \tag{17}$$

If K^* is not an interior solution of the region where $K^* \leq \frac{(1-\alpha)\theta}{2(1-\gamma)}$, the optimal capacity is given by the boundary $\theta\frac{(1-\alpha)}{2(1-\gamma)}$. Thus,

$$K^*(\theta) = \min\left[K^*, \theta\frac{(1-\alpha)}{2(1-\gamma)}\right]. \tag{18}$$

The investment threshold θ^* is implicitly determined by

$$\begin{aligned} B_2\theta^{*\beta_2}(\beta_1-\beta_2) &+ \frac{\theta^* K_F^*(\theta^*)}{r-\mu}(\beta_1-1) \\ &- \beta_1\frac{K_F^*(\theta^*)^2}{r} - \beta_1 c_F K_F^*(\theta^*) = 0. \end{aligned} \tag{19}$$

From these two possibilities the firm chooses the one that gives the higher expected value of the project discounted back to an initial demand intercept level θ_0, which is given by $\left(\frac{\theta_0}{\theta_F^*}\right)^{\beta_1} V\left(\theta_F^*, K_F^*(\theta_F^*)\right)$.

2.2 Dedicated Capacity

Consider a firm that must decide about investment in dedicated capacity. Dedicated capacity can produce only one product. It is assumed that if the firm wants to produce both products, it invests in the two capacities at the same time. The firm must choose when to invest and the optimal capacity. It has two options. It can invest in both dedicated production capacities and thus produce both A and B. Alternatively, if product B is not sufficiently profitable the firm can invest in a dedicated production facility for product A. The unit costs of capacity are assumed to be equal for both products, i.e.,

$$c_{D_A} = c_{D_B} =: c_D. \tag{20}$$

A higher unit cost for one of the products would result in a proportionally lower capacity investment for this product. With assumption (20) I exclude the possible impact of cost differences for the two products in order to concentrate on the effect of product profitability and substitutability, one of the main aspects of this paper. The firm will choose the option with the higher expected project value. After investment the firm has to produce at full capacity indefinitely.

If the firm invests only in product A, its profit is given by

$$\Pi = (\theta - q_A)q_A. \tag{21}$$

Given the capacity clearance constraint, $q_A = K_{D,A}$, the profit flow can be rewritten as a function of the dedicated capacity $K_{D,A}$:

$$\Pi = (\theta - K_{D,A})K_{D,A}. \tag{22}$$

Familiar steps lead to the following value for the investment project:

$$V(\theta, K_{D,A}) = \frac{\theta K_{D,A}}{r-\mu} - \frac{K_{D,A}^2}{r}. \tag{23}$$

If the firm invests in both A and B, its profit $\Pi = p_A q_A + p_B q_B$ is maximized w.r.t. the output rates q_A and q_B. Given the capacity clearance constraints for the two products, $q_A = K_{D,A}$ and $q_B = K_{D,B}$, the profit flow is given by

$$\Pi(\theta, K_{D,A}, K_{D,B}) = (\theta - K_{D,A})K_{D,A} + (\alpha\theta - K_{D,B})K_{D,B} - 2\gamma K_{D,A}K_{D,B}. \tag{24}$$

Familiar steps lead to the following project value:

$$V(\theta, K_{D,A}, K_{D,B}) = \frac{\theta}{r-\mu}\left[K_{D,A} + \alpha K_{D,B}\right] - \frac{K_{D,A}^2 + 2\gamma K_{D,A}K_{D,B} + K_{D,B}^2}{r}. \tag{25}$$

The optimal capacity for every relevant value of θ is derived by maximizing the project value minus the investment cost $c_D(K_{D,A} + K_{D,B})$ for a given demand intercept level θ. If $K_{D,i}$ $(i = A, B)$ is negative, the optimal capacity is replaced by the boundary solution $K_{D,i}^*(\theta) = 0$. Given the optimal capacity for all relevant demand levels, the optimal investment threshold θ^* can be derived. The following proposition provides expressions for the threshold θ^* and the corresponding capacity $K_D^*(\theta^*)$ in the two cases. The optimal investment decision corresponds to the investment strategy that provides the larger expected project value for the firm.

Proposition 2 *The firm has two possible investment policies for dedicated capacity:*

1. *If it is optimal for the firm to invest in dedicated production capacity for both products the optimal capacities for products A and B are given by*

$$K_{D,A}^*(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta}_A, \\ \frac{\theta r}{2(r-\mu)}\frac{(1-\alpha\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \theta > \hat{\theta}_A, \end{cases} \tag{26}$$

$$K_{D,B}^*(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta}_B, \\ \frac{\theta r}{2(r-\mu)}\frac{(\alpha-\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \theta > \hat{\theta}_B, \end{cases} \tag{27}$$

where $\hat{\theta}_A = \frac{c_D(r-\mu)(1-\gamma)}{(1-\alpha\gamma)}$ *and* $\hat{\theta}_B = \frac{c_D(r-\mu)(1-\gamma)}{(\alpha-\gamma)}$. *The total optimal dedicated capacity is given by*

$$K_D^*(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta}_A, \\ \frac{\theta r}{2(r-\mu)}\frac{(1-\alpha\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \hat{\theta}_A < \theta < \hat{\theta}_B, \\ \theta\frac{r}{2(r-\mu)}\frac{(1+\alpha)}{(1+\gamma)} - c_D\frac{r}{(1+\gamma)} & \text{for } \hat{\theta}_B < \theta. \end{cases} \tag{28}$$

(29)

The investment thresholds are given by

$$\theta_{D,I}^* = \frac{\beta_1 c_D(r-\mu)(1+\gamma-2\gamma^2)}{\beta_1(1+\alpha\gamma) - 2(1+(\beta_1-1)\gamma^2)} \tag{30}$$

for $\hat{\theta}_A < \theta < \hat{\theta}_B$, and

$$\theta_{D,II}^* = \frac{c_D(r-\mu)(1+\alpha)(1-\gamma)}{(1-2\alpha\gamma+\alpha^2)}\left(\frac{\beta_1-1}{\beta_1-2}\right) + \frac{2(r-\mu)^2(1-\gamma^2)}{r(1-2\alpha\gamma+\alpha^2)(\beta_1-2)} \sqrt{\frac{r^2c_D^2(1+\alpha)^2(\beta_1-1)^2}{4(r-\mu)^2(1+\gamma)^2} - \frac{r^2c_D^2(1-2\alpha\gamma+\alpha^2)\beta_1(\beta_1-2)}{2(r-\mu)^2(1-\gamma^2)(1+\gamma)}} \tag{31}$$

for $\theta > \hat{\theta}_B$.

2. *If it is more profitable for the firm to invest only in product A, the optimal capacity for A is given by*

$$K^*_{D,A}(\theta) = \frac{r}{2(r-\mu)}\theta - \frac{r}{2}c_D. \tag{32}$$

The investment threshold is given by

$$\theta^*_D = \left(\frac{\beta_1}{\beta_1 - 2}\right)(r-\mu)c_D. \tag{33}$$

From these two possibilities the firm chooses the one that gives the higher expected value of the project discounted back to an initial time with demand intercept level θ_0*, which is given by* $\left(\frac{\theta_0}{\theta^*_D}\right)^{\beta_1} V\left(\theta^*_D, K^*_D(\theta^*_D)\right)$.

3 Results

This section presents results for investment in flexible and dedicated capacity independently. The optimal investment strategy of a firm considering both flexible and dedicated capacity will be analyzed in Section 4.

3.1 Flexible-Capacity Investment

As shown in Section 2.1, the flexible firm can invest in θ-region I, i.e., $\theta \in [0, \frac{2(1-\gamma)}{1-\alpha}K_F)$, where the firm sets an upper bound on the output and uses the new capacity to produce both products immedately after investment, or invest in the second θ-region, i.e., $\theta \in [\frac{2(1-\gamma)}{1-\alpha}K_F, \infty)$. In region II the firm invests in flexible capacity that is fully used to produce the more profitable A. After the investment the firm can adapt the relative production volumes to the changing demand. Under low demand it will make use of the downside potential to produce both products in order to increase the total market size. Under high demand the firm will use the full capacity to produce the more profitable A.

Figure 1 shows an example of two highly substitutable products with substitutability $\gamma = 0.8$, assuming a substantial difference in the profitability of the two products. Product B is much less profitable than A with profitability $\alpha = 0.2$. The other assumed parameter values are $\mu = 0.02$, $\sigma = 0.1$, $r = 0.1$, and $c_F = 100$. Solving Eqs. (14) and (17) gives the optimal capacity for the two regions. Comparing the expected values of the investment for the two regions we see that it is optimal to invest in region II at the investment trigger $\theta^* = 21.147$, provided that the initial θ-value lies below this θ^*. In particular, the firm invests immediately if the current value of θ exceeds θ^*; otherwise it waits until θ becomes equal to θ^*. The optimal acquired capacity is $K^*(\theta^*) = 8.22$. After the investment the firm can adapt the relative production volumes according to the changing demand to receive the highest possible profit. For this example the firm will continue producing only A unless the demand drops drastically, i.e., below $\hat{\theta} = 4.11$. Since the two products are good substitutes in the market but producing B results in significantly less profit for the firm, it is optimal for a wide range of demands to continue producing only the more profitable A. For very

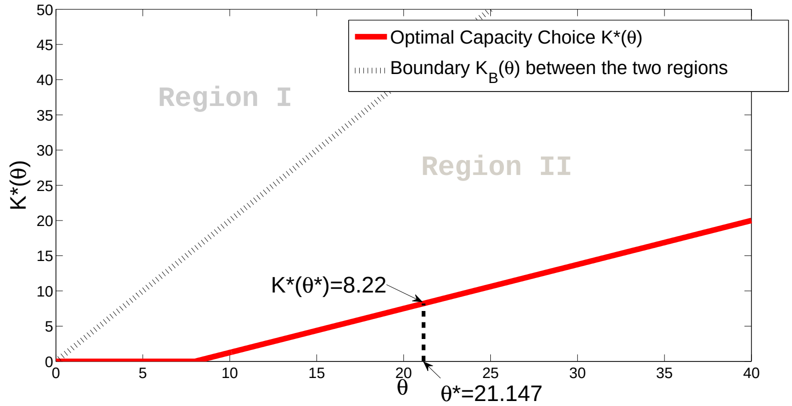

Figure 1: Optimal investment capacity as function of demand intercept θ with $\alpha = 0.2$, $\gamma = 0.8$. Region I is the region where the capacity for a specific demand realization is used to produce both products. Region II is the area where it is optimal for the firm to use all the capacity to produce the more profitable A. (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, and $c_F = 100$.)

low demand the firm can profit from the downside potential and avoid overcapacity by increasing the total market size including the demand for product B.

If we choose a relatively low substitutability ($\gamma = 0.2$) but a high profitability for product B ($\alpha = 0.8$), we significantly increase the downside potential. In fact, it is optimal for the firm to invest in capacity at the investment threshold $\theta_F^* = 22.669$. For this parameter choice the investment occurs in region II, which means that the firm uses the new capacity to produce both products. Figure 2 (which illustrates this example) shows the optimal capacity as a function of the demand intercept θ. The figure shows that, in contrast to the previous example, the capacity function $K^*(\theta)$ switches at $\hat{\theta_S} = 9.902$ from investment in region II to investment in region I. Compared to the previous example, the flexible capacity is valuable for a firm selling two almost equally profitable products with low substitutability. The firm purchases significantly more capacity, $K^*(\theta^*) = 12.94$.

Figure 3 illustrates the advantage of flexible production capacity for a firm selling two almost equally profitable products with low substitutability. The upper plot shows a simulation of the demand intercept θ with a drift rate of $\mu = 0.02$ and volatility $\sigma = 0.1$ for a period of 10 years, i.e., $t \in [0, 10]$. The firm will invest as soon as the demand intercept reaches $\theta_F^* = 22.669$ for the first time, which is (for this simulation) after 1.6 years. The lower plot shows the optimal production decision after the investment. Flexible capacity allows the firm to adapt the relative production volume of the two products to obtain the highest possible profit given its capacity constraint of $K_F^*(\theta^*) = 12.94$. For low demand the firm uses the capacity to produce both products. If the demand rises above a certain threshold, i.e., when the demand intercept reaches $\hat{\theta} = 103.52$,

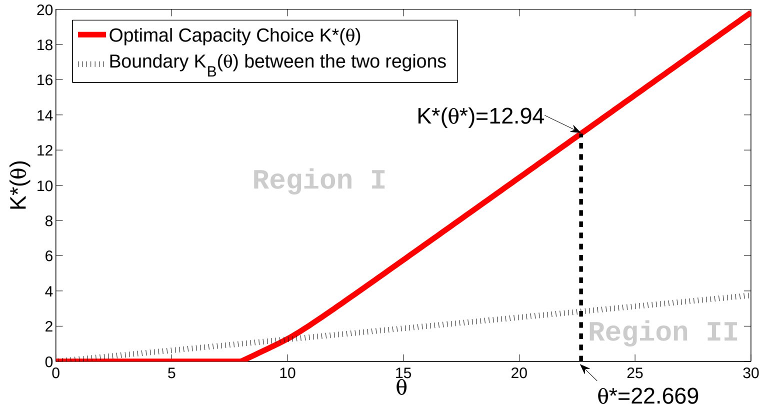

Figure 2: Optimal investment capacity as function of demand intercept θ with $\alpha = 0.8$, $\gamma = 0.2$. (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, and $c_F = 100$.)

it is most profitable to use the full capacity (i.e., $K_F^*(\theta^*) = 12.94$) to produce A. Being able to adapt the relative production volumes allows the firm to avoid over- and under-capacity for a wide range of possible demands.

I now analyze the effect of demand variability on the flexible investment decision. Four specific "extreme" cases that arise from different combinations of product profitability and substitutability are compared. *Case H-L* is a combination of two similarly profitable products with low substitutability (parameter values $\alpha = 0.9$ and $\gamma = 0.1$). *Case H-H* is a combination of two highly substitutable and almost equally profitable products (parameter values $\alpha = 0.9$ and $\gamma = 0.9$). *Case L-H* is a combination of two products that are highly substitutable where one is significantly less profitable than the other (parameter values $\alpha = 0.1$ and $\gamma = 0.9$). *Case L-L* is a combination of two products with low substitutability and a significant difference in profitability (parameter values $\alpha = 0.1$ and $\gamma = 0.1$).

The impact of demand variability on the optimal capacity and investment threshold is illustrated in Fig. 4. The parameter values are $r = 0.1$, $\mu = 0.02$, and $c_F = 100$; these values form the base case for the examples in the remainder of the paper. The figure confirms the widely accepted result that higher uncertainty increases capacity but delays investment. When uncertainty rises, a higher demand is needed before it is optimal to invest. This effect arises partly because capacity increases with uncertainty, and partly because of the real option result that in a more uncertain economic environment the firm has a greater incentive to wait for more information before investing (see Dixit and Pindyck (1994)).

Figure 4 shows that the capacity is higher for the firm selling two products with low substitutability and a high profitability difference. The difference in capacity between Case L-H and the other cases becomes more significant for higher uncertainty. The capacity difference of the four cases is relatively small when the profit

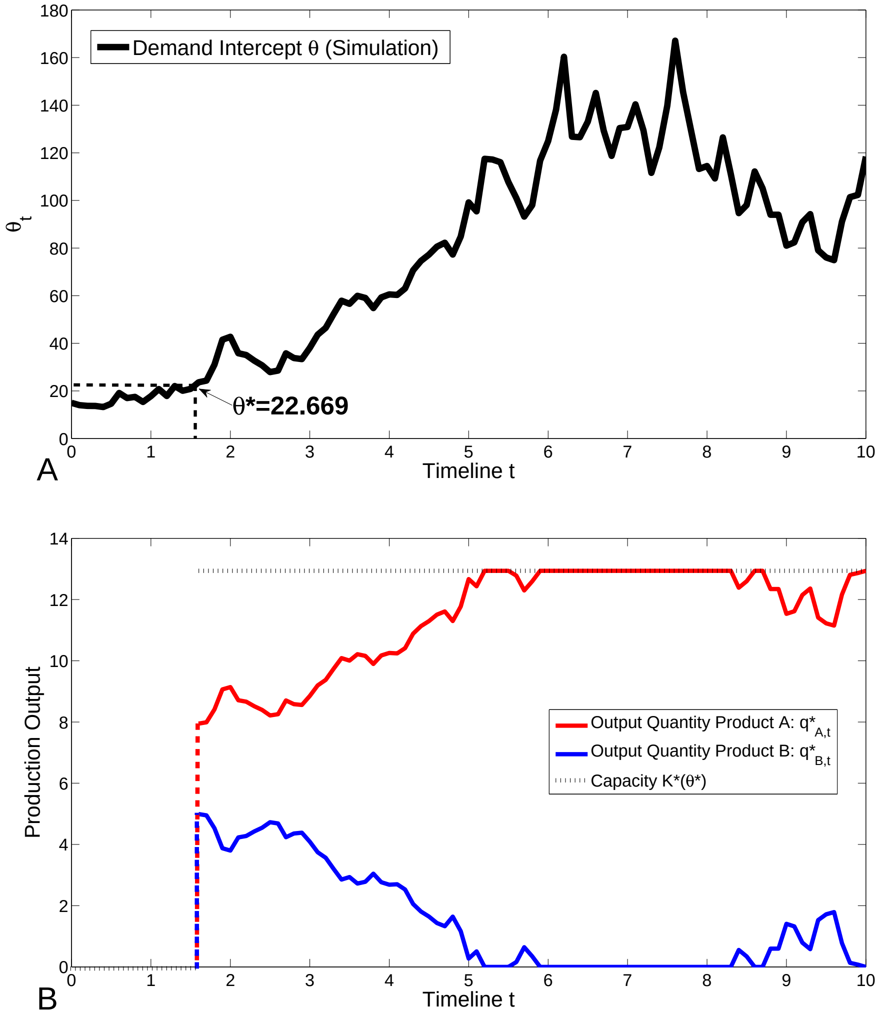

Figure 3: Production timeline for simulation of demand intercept θ. Panel A: Simulated demand intercept process θ_t plotted against timeline. Panel B: Optimal production output of A and B over time period $t \in [0, 10]$. (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, and $c_F = 100$.)

Table 1: Optimal investment strategy for changing profitability α and fixed substitutability $\gamma = 0.1$. The expected profit is discounted back to an initial demand value $\theta_0 = 10$ for comparison purposes. The left panel shows the results for uncertainty $\sigma = 0.1$, the right panel for $\sigma = 0.2$. (Parameter values: $r = 0.1$, $\mu = 0.02$, and $c_F = 100$.)

α	θ_F	K_F^*	region	Π_F	θ_F	K_F^*	region	Π_F
0.1	21.19	8.28	II	60.75	80.55	51.17	I	204.14
0.2	21.25	8.36	II	60.83	81.91	54.13	I	208.05
0.3	21.42	8.61	I	61.04	83.23	57.64	I	213.72
0.4	21.90	9.35	I	61.75	84.28	61.48	I	221.67
0.5	22.56	10.61	I	63.64	84.83	65.41	I	232.46
0.6	23.06	12.06	I	67.42	84.67	69.12	I	246.71
0.7	23.15	13.33	I	73.58	83.68	72.34	I	265.04
0.8	22.81	14.25	I	82.50	81.84	74.83	I	288.15
0.9	22.11	14.78	I	94.60	79.18	76.43	I	316.85

difference is 50% (see Table 3). When the profit difference is approximately constant, the capacity difference grows more than proportionally. This confirms the intuition that the high capacity is not driven only by the general argument of "investing later in more capacity." It is strengthened by the high value of product-flexible capacity for a firm that produces two almost equally profitable products with low substitutability, which increases its willingness to invest in capacity. Panel A of Fig. 3 shows the demand range over which the firm will benefit from product-flexible capacity. It will produce both products for a wide range ($\theta \in [0, \hat{\theta})$) and suspend the production of product B only under extremely high demand.

Table 1 shows the effect of profitability on the optimal investment strategy with the substitutability rate constant and low ($\gamma = 0.1$). Observe that the optimal capacity and the expected profit increase with the profitability of product B. The nonmonotonic effect of α on the investment threshold is striking. For a graphical illustration of the nonmonotonic behavior of investment timing see Fig. 5. This result is driven by two contradictory effects: on the one hand the firm invests later in more capacity while on the other hand a higher project value leads the firm to invest earlier. The capacity effect is stronger for low profitability parameters while the effect of the higher project value dominates for two almost equally profitable products.

Figure 5 shows the optimal investment thresholds when the substitutability of the two products is low and the profitability of product B ranges from low (0.1) to high (0.9). The optimal investment threshold increases with α for low values of α and decreases for high values. This effect is stronger for high demand

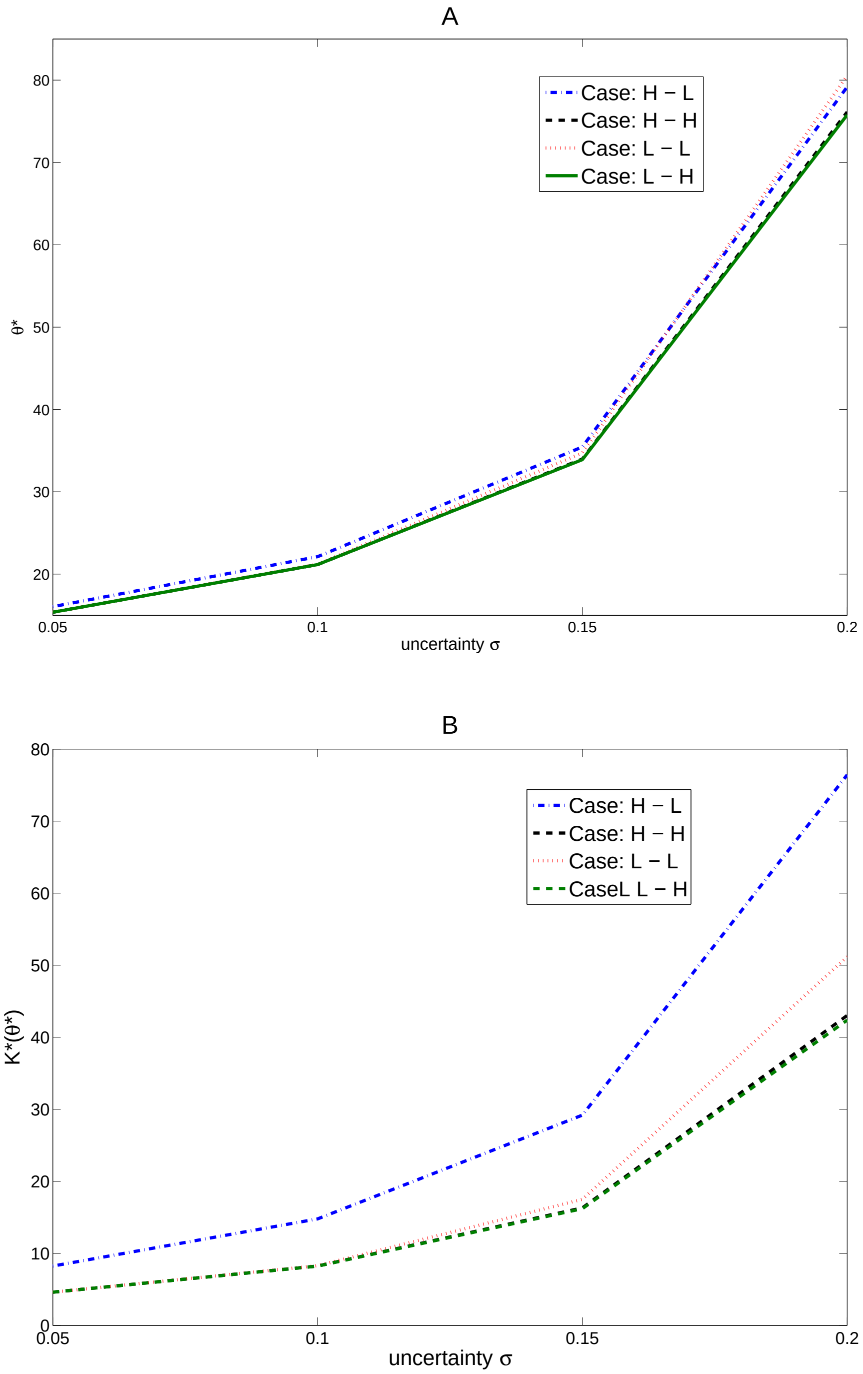

Figure 4: Optimal investment strategy for four cases: Case H-L: $\alpha = 0.9$, $\gamma = 0.1$, Case H-H: $\alpha = 0.9$, $\gamma = 0.9$, Case L-L: $\alpha = 0.1$, $\gamma = 0.1$, Case L-H: $\alpha = 0.1$, $\gamma = 0.9$. Panel A: Optimal investment threshold θ^* as function of demand volatility σ. Panel B: Optimal capacity $K^*(\theta^*)$ as function of volatility σ. (Parameter values: $r = 0.1$, $\mu = 0.02$, and $c_F = 100$.)

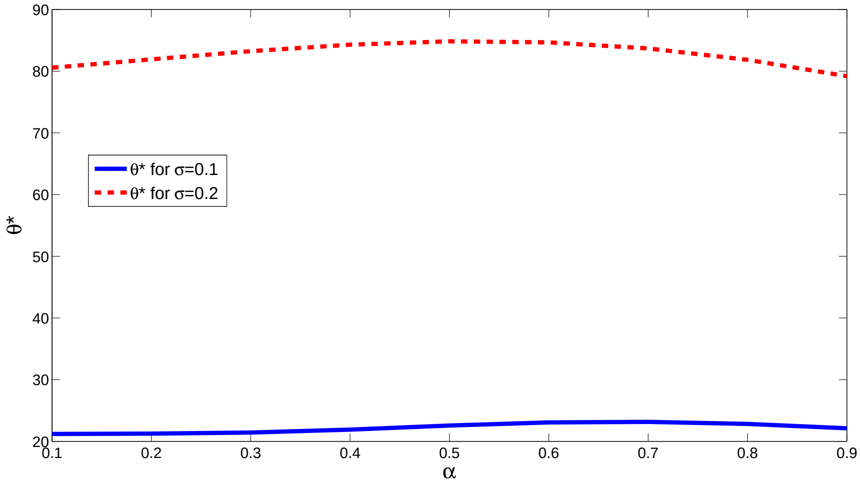

Figure 5: Optimal investment threshold as function of profitability α. (Parameter values: $\gamma = 0.1$, $r = 0.1$, $\mu = 0.02$, and $c_F = 100$.)

volatility. The capacity and expected profit both increase with α.

3.2 Dedicated Capacity Investment

It is surprising that in most cases the firm will purchase dedicated capacity only for the more profitable product. For the four cases, Table 2 shows the optimal investment thresholds for specific parameter values. For the case of almost equally profitable products with low substitutability the firm purchases significantly more dedicated capacity for A than for the (slightly) less profitable B. For all the other cases the firm invests only in product A. To gain intuition into this result, note that in the latter three cases the demand would have to become very low for the firm to gain from the possibility of increasing the total market size by producing the less profitable B. In most situations it can gain a higher profit by satisfying only the demand for product A. The threat of a possible overcapacity for product B dominates the potential to increase the total market demand by producing both products under low demand.

Only the firm with a combination of two similarly profitable products with low substitutability can profit from this downside potential over a wide demand range.

4 Value of Flexibility

One of the main objectives of this paper is to quantify the value of flexible capacity. To derive the flexibility value, the situation in which a firm relies on at most two dedicated capacities rather than on one flexible

Table 2: Investment strategies for dedicated capacity investment for the four cases (description in Fig. 4). (Parameter values: $L = 0.1$, $H = 0.9$, $r = 0.1$, $\mu = 0.02$, and $c_D = 100$.)

$\alpha = H$, $\gamma = L$

σ	θ^*	$K^*_{D,A}(\theta^*)$	$K^*_{D,B}(\theta^*)$	$K^*_D(\theta^*)$
0.05	16.0587	4.6802	3.56501	8.24522
0.1	22.1392	8.17342	6.63597	14.8094
0.15	35.5252	15.8636	13.3966	29.2602
0.2	79.4644	41.1065	35.5881	76.6946

$\alpha = H$, $\gamma = H$; $\alpha = L$, $\gamma = L$; and $\alpha = L$, $\gamma = H$

σ	θ^*	$K^*_{D,A}(\theta^*)$
0.05	15.3644	4.60274
0.1	21.1472	8.21699
0.15	33.8987	16.1867
0.2	75.7771	42.3607

capacity is considered as a benchmark. The optimal investment strategies for flexible and dedicated capacity investment are compared, assuming that the unit investment costs are equally high, i.e., $c_D = c_F =: c$. The value of flexibility is therefore given by the difference in the expected profit of the investment strategies.

To compare two investment strategies that have different timings, we must compare the discounted expected project values. Assuming the optimal investment thresholds derived in Section 3.2, the value of flexibility is:

$$V_f = \left(\frac{\theta_0}{\theta^*_F}\right)^{\beta_1} V\left(\theta^*_F, K^*_F(\theta^*_F)\right) - \left(\frac{\theta_0}{\theta^*_D}\right)^{\beta_1} V\left(\theta^*_D, K^*_D(\theta^*_D)\right), \tag{34}$$

where the expected project values are discounted back to the beginning of the time period with initial demand intercept θ_0.

Table 3 shows the value of flexibility (V_f) for the example presented in the previous section. The discounted expected project values, denoted Π_i for $i = D, F$, are discounted back to the beginning of the time period, where the demand is given by $\theta_0 = 10$. Furthermore, the table gives the ratio of the expected profit of dedicated investment to that of flexible investment, i.e., $\frac{\Pi_D}{\Pi_F}$.

Table 3 shows that the value of flexibility is most significant when uncertainty is high, substitutability is low, and the profit levels of the two products are substantially different (see $\alpha = L$, $\gamma = L$). When demand uncertainty $\sigma = 0.2$, substitutability $\gamma = 0.1$, and B has a low profitability compared to A, the value of

Table 3: Optimal profits for flexible and dedicated investment, discounted back to the initial demand level $\theta_0 = 10$; comparison of four cases (description in Fig. 4). Π_F (Π_D) is discounted expected profit of flexible (dedicated) capacity; V_f is value of flexibility. (Parameter values: $L = 0.1$, $H = 0.9$, $r = 0.1$, $\mu = 0.02$, and $c = 100$.)

σ	θ_F^*	K_F^*	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}\%$
$\alpha = H$, $\gamma = L$										
0.05	16.05	8.23	52.67	16.06	4.68	3.57	8.25	52.59	0.09	99.83
0.1	22.11	14.78	94.60	22.14	8.17	6.64	14.81	94.38	0.	99.77
0.15	35.44	29.18	172.06	35.53	15.86	13.40	29.26	171.49	0.57	99.67
0.2	79.18	76.43	316.85	79.46	41.11	35.59	76.69	315.41	1.44	99.55
$\alpha = H$, $\gamma = H$										
0.05	15.36	4.60	35.30	15.36	4.60	0	4.60	35.30	0	100
0.1	21.15	8.22	60.69	21.15	8.22	0	8.22	60.69	0.01	99.99
0.15	33.97	16.30	107.42	33.90	16.19	0	16.19	107.25	0.17	99.84
0.2	76.14	43.02	194.67	75.78	42.36	0	42.36	193.74	0.93	99.52
$\alpha = L$, $\gamma = L$										
0.05	15.36	4.60	35.30	15.36	4.60	0	4.60	35.30	0	100
0.1	21.19	8.28	60.75	21.15	8.22	0	8.22	60.69	0.07	99.89
0.15	34.71	17.49	108.98	33.90	16.19	0	16.19	107.25	1.73	98.41
0.2	80.55	51.17	204.14	75.78	42.36	0	42.36	193.74	**10.40**	**94.90**
$\alpha = L$, $\gamma = H$										
0.05	15.36	4.60	35.30	15.36	4.60	0	4.60	35.30	0	100
0.1	21.15	8.22	60.69	21.15	8.22	0	8.22	60.69	0	100
0.15	33.90	16.19	107.25	33.90	16.19	0	16.19	107.25	0	100
0.2	75.78	42.37	193.75	75.78	42.36	0	42.36	193.74	0.01	99.99

flexibility is substantially higher than for the other cases. Flexibility pays off especially in this case because it allows the firm to avoid over-capacity by increasing the market size by producing the less profitable product under low demand, while a dedicated firm is restricted to the more profitable product. Because the optimal investment capacity for the flexible firm is significantly higher ($K_F^* = 51.17$) than that for the dedicated firm ($K_D^* = 42.36$), the flexible firm is also less threatened by under-capacity in high-demand periods.

Table 4 shows a wider range of profitability and substitutability combinations. Given the previous result, that flexibility is most valuable under high demand uncertainty, σ is set to 0.2. The table shows three cases: low ($\gamma = 0.1$), medium ($\gamma = 0.5$), and high ($\gamma = 0.9$) substitutability. The profitability ranges from $\alpha = 0.1$ to $\alpha = 0.9$. As expected, the value of flexibility is highest for low substitutability and low profitability, specifically $\alpha = 0.1, 0.3$, and 0.5. As α increases from 0.1 to 0.5, the capacity difference between the flexible and the dedicated case ($K_F^* - K_D^*$) is high and decreases with α. This indicates that the advantage of flexible capacity under high demand decreases with α. On the other hand, the demand range for which the firm will produce both products, i.e., $\hat{\theta}$, increases with α, which allows the firm to profit more from the downside potential of flexible capacity. The value of flexibility is highest for $\alpha = 0.3$.

These results are derived assuming that the unit costs of dedicated and flexible capacity are equally high. However, flexible capacity is usually more expensive than dedicated capacity. Increasing the unit cost of flexible capacity to the level at which the expected value of flexible investment (Π_F) is equal to the expected value of dedicated investment (Π_D) shows how much more a firm is willing to pay for flexible capacity. The results are striking. Figure 6 shows the unit cost of flexible capacity for which $\Pi_F = \Pi_D$ as a function of the profitability when substitutability is low ($\gamma = 0.1$) and demand uncertainty is high ($\sigma = 0.2$). For a large difference in the profitability levels of the two products, i.e., $\alpha = 0.1$, the unit cost of flexible capacity can increase by 25% compared to the dedicated unit cost. When $\alpha = 0.3$ the firm would pay 40% more for flexible capacity.

The relatively low value of flexibility in the other cases is partly driven by the fact that the firm can freely decide the optimal capacity investment and can choose to ignore the demand for the less profitable B by purchasing just one dedicated capacity. Often, however, firms are must decide whether or not to invest in flexible capacity when they are currently producing with dedicated capacity. I consider this in the following section.

5 Incentive to Change from Dedicated to Flexible Capacity

Assume that a firm with two dedicated capacities for its products, A and B, can switch to flexible capacity by paying an investment cost of $c_F K_F$. In the following I will analyze for which specific product mixes and parameter ranges the firm should switch. The dedicated capacities are given by $K_{D,A}$ and $K_{D,B}$. Let

Table 4: Optimal profits of flexible and dedicated investment discounted back to initial demand level $\theta_0 = 10$. Π_F (Π_D) is discounted expected profit of flexible (dedicated) capacity; V_f is value of flexibility. $\hat{\theta}$ is the boundary at which it is optimal for the firm to change from producing both products to producing only A. (Parameter values: $\sigma = 0.2$, $r = 0.1$, $\mu = 0.02$, and $c = 100$.)

α	θ_F^*	K_F^*	$\hat{\theta}$	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}$
$\gamma = L = 0.1$											
0.1	80.6	51.2	102	204.1	75.8	42.4	0	42.4	193.7	10.4	94.9
0.3	83.2	57.6	148.1	213.7	82.3	45.8	5.8	51.7	196.6	17.1	92.0
0.5	84.8	65.4	235.5	232.5	87.5	47.9	17.6	65.5	217.1	15.4	93.4
0.7	83.7	72.3	433.8	265.0	85.49	45.7	27.8	73.5	256.6	8.4	96.8
0.9	79.2	76.4	1375	316.9	79.5	41.1	35.6	76.7	315.4	1.5	99.5
$\gamma = M = 0.5$											
0.1	76.3	43.3	48.1	195.0	75.8	42.4	0	42.4	193.7	1.3	99.3
0.3	76.3	43.76	62.5	196.0	75.8	42.4	0	42.4	193.7	2.3	98.9
0.5	77.9	46.3	92.6	198.9	75.8	42.4	0	42.4	193.7	5.2	97.4
0.7	79.7	51.0	170	208.6	80.6	40.3	10.1	50.4	200.8	7.8	96.3
0.9	78.6	55.6	556.0	234.7	79.0	32.9	23.0	55.9	232.7	2.0	99.1
$\gamma = H = 0.9$											
0.1	75.8	42.4	9.4	193.7	75.8	42.4	0	42.4	193.7	0	100
0.3	75.8	42.4	12.1	193.8	75.8	42.4	0	42.4	193.7	0.1	99.9
0.5	75.8	42.4	17.0	193.8	75.8	42.4	0	42.4	193.7	0.1	99.9
0.7	75.8	42.4	28.3	193.8	75.8	42.4	0	42.4	193.7	0.1	99.9
0.9	76.1	43.0	86.0	194.7	75.8	42.4	0	42.4	193.7	1.0	99.5

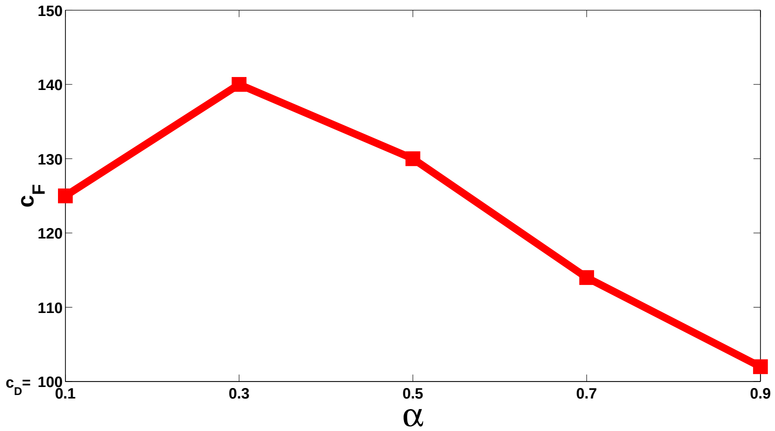

Figure 6: c_F is the unit cost of flexible capacity that the firm is willing to pay so that the expected project values for flexible and dedicated capacity investment are equal, i.e., $\Pi_F = \Pi_D$, assuming that $c_D = 100$. (Parameter values: $\gamma = 0.1$, $r = 0.1$, $\mu = 0.02$, $\sigma = 0.2$, and $\theta_0 = 10$.)

$V_D(\theta, K_{D,A}, K_{D,B})$ be the expected value of the firm producing with the dedicated capacities starting with a demand intercept θ. Similarly, the value of the firm producing with flexible capacity is given by $V_F(\theta, K_F)$, assuming a flexible capacity of K_F. The expressions for V_F and V_D are equal to the project values derived in Sections 3.2 and 2.2, respectively.

The threshold at which it is optimal to switch from dedicated to flexible capacity must satisfy the standard pair of conditions for optimality, which are the value matching

$$\left[V_D(\theta, K_{D,A}, K_{D,B}) + D_1\theta^{\beta_1} + D_2\theta^{\beta_2}\right]\Big|_{\theta=\theta^*} = \left[V_F(\theta, K_F) - c_F K_F\right]\Big|_{\theta=\theta^*} \tag{35}$$

and the smooth pasting condition

$$\left[\frac{\partial}{\partial\theta}V_D(\theta, K_{D,A}, K_{D,B}) + \beta_1 D_1\theta^{\beta_1-1} + \beta_2 D_2\theta^{\beta_2-1}\right]\Bigg|_{\theta=\theta^*} = \left[\frac{\partial}{\partial\theta}V_F(\theta, K_F)\right]\Bigg|_{\theta=\theta^*}. \tag{36}$$

$D_1\theta^{\beta_1}$ and $D_2\theta^{\beta_2}$ are option values. Specifically, $D_1\theta^{\beta_1}$ is the value of the option to purchase flexible capacity and increase profit if θ increases. $D_2\theta^{\beta_2}$ is the value of the option to increase profit at a later time by installing flexible capacity in case demand decreases. As shown in the previous section, flexibility is especially valuable in very low and very high demand regions. Therefore, we know that the function $[V_F - V_D](\theta)$ is convex in the demand intercept θ.

There are two effects that may increase the firm's profit when it adopts flexible technology. On the one hand, it can optimally adapt the product mix to the demand. This means that it can use all the capacity to produce the more profitable product if demand is high. If demand is low it can increase the total market demand by producing both products and optimally adapting the output rates. On the other hand, the firm can also gain by investing in flexible capacity because it can update the optimal total capacity. This is because it has more up-to-date information about the economic environment than it had when the dedicated capacity was installed. If demand is high the firm can gain by increasing the total capacity. If demand is low it can profit by reducing the total capacity. Recall that downscaling can be valuable because the firm must use the full capacity for production (the capacity clearance assumption). $D_2\theta^{\beta_2}$ is therefore in part the value of a disinvestment option.

However, I want to concentrate on the additional value of flexibility in comparison to dedicated capacity in high-demand regions. Therefore, I assume that $K_F \geq K_{D,A} + K_{D,B}$, which excludes the possibility of disinvestment. Furthermore, I rule out scenarios where it might be favorable to invest in flexible capacity with regard to its downside potential. Therefore, I set a lower bound on the investment cost, so that the value of flexibility does not exceed the cost of investment in low-demand regions. Assuming that $c_F > \hat{c}_F = \left[\frac{1-\gamma}{2r}\frac{(K_{D,A}-K_{D,B})^2}{(K_{D,A}+K_{D,B})}\right]$, it follows that $D_2 = 0$. For the derivation of this bound see Appendix D. Therefore, the value matching and smooth pasting conditions are given by

$$\left[V_D(\theta, K_{D,A}, K_{D,B}) + D_1\theta^{\beta_1}\right]\Big|_{\theta=\theta^*} = \left[V_F(\theta, K_F) - c_F K_F\right]\Big|_{\theta=\theta^*}, \tag{37}$$

$$\left[\frac{\partial}{\partial\theta}V_D(\theta, K_{D,A}, K_{D,B}) + \beta_1 D_1\theta^{\beta_1-1}\right]\Bigg|_{\theta=\theta^*} = \left[\frac{\partial}{\partial\theta}V_F(\theta, K_F)\right]\Bigg|_{\theta=\theta^*}. \tag{38}$$

The following proposition states the implicit equations for the investment trigger for both regions.

Proposition 3 *Consider a firm's possible switch to flexible capacity K_F while currently producing with two dedicated capacities $K_{D,A}$ and $K_{D,B}$. Assuming that $K_F \geq K_{D,A}+K_{D,B}$, the optimal investment trigger is implicitly determined by the following equations:*

$$\begin{aligned}\left(\frac{\beta_1-2}{\beta_1}\right)a_1\theta^{*2} \quad &+ \quad \left(\frac{\beta_1-1}{\beta_1}\right)a_2\theta^* K_F + a_3K_F^2 - \left(\frac{\beta_1-1}{\beta_1}\right)\frac{\theta^*}{r-\mu}\left[K_{D,A}+\alpha K_{D,B}\right] \\ &+ \quad \frac{K_{D,A}^2 + 2\gamma K_{D,A}K_{D,B} + K_{D,B}^2}{r} = c_F K_F\end{aligned} \tag{39}$$

for region I (i.e., $\theta^* < \frac{2(1-\gamma)K_F}{1-\alpha}$) and

$$\begin{aligned}\left(\frac{\beta_1-\beta_2}{\beta_1}\right)B_2\theta^{*\beta_2} \quad &+ \quad \left(\frac{\beta_1-1}{\beta_1}\right)\frac{\theta^* K_F}{r-\mu} - \frac{K_F^2}{r} - \frac{\theta^*}{r-\mu}\left[K_{D,A}+\alpha K_{D,B}\right] \\ &+ \quad \frac{K_{D,A}^2 + 2\gamma K_{D,A}K_{D,B} + K_{D,B}^2}{r} = c_F K_F\end{aligned} \tag{40}$$

for region II (i.e., $\theta^* \geq \frac{2(1-\gamma)K_F}{1-\alpha}$).

Table 5 presents the optimal investment thresholds for different product combinations. These results are derived assuming $r = 0.1$, $\mu = 0.02$, the unit cost of flexible capacity $c_F = 10$, $K_{D,A} = 8$, and $K_{D,B} = 6$. The demand volatility is $\sigma = 0.05$, 0.1, 0.15, or 0.2. As in the previous sections, $\alpha = L$ means that the difference in the profitability of the two products is high, i.e., A is much more profitable than B. If $\alpha = H$

the two products are almost equally profitable. For the example presented in Table 5, $L = 0.1$ and $H = 0.9$. The same numerical values are chosen for the substitutability γ.

Regarding the optimal timing for the switch from dedicated to flexible capacity, the investment threshold differs substantially for different product combinations. This is surprising, since the product demand is assumed to be perfectly positively correlated. The firm invests earlier in flexible capacity if the profitability of the products is substantially different (see $\gamma = L$), while for a mix of two almost equally profitable products, a higher demand threshold is required. For example, if the demand uncertainty is $\sigma = 0.1$, the optimal investment threshold is $\theta^* = 156.27$ for low substitutability and $\theta^* = 45.63$ for high substitutability if the profit level of both products is high. If the products differ substantially in profitability, the optimal investment threshold is substantially lower: $\theta^* = 17.36$ for low substitutability and $\theta^* = 5.07$ for high substitutability. Since product A is much more profitable when α is low, the firm could increase its profit substantially by increasing the output rate for this product. Therefore, the value of waiting is dominated by the "cost" of lost profit if the firm continues producing too much of B in relation to the more profitable A.

A firm that produces two products with low substitutability and similar profitability will switch to flexible capacity, keeping the total capacity the same, only under high demand. The firm does not have a strong incentive to switch because $K_{D,A} = 8$ and $K_{D,B} = 6$ are a good choice for this product combination; therefore, the switch does not greatly increase profit. However, if the firm could increase its total capacity via investment in flexible capacity, this would accelerate the investment substantially to $\theta^* = 37.76$ for $K_F = 15$ and $\theta^* = 29.84$ for $K_F = 16$. The favorable product combination results in a large option value for increased total capacity, and therefore the firm invests earlier if the total capacity can be increased.

If $\alpha = L$, i.e., B is much less profitable than A, the opposite holds. The possibility of increasing the total capacity delays the investment.

There are two contradictory effects that either accelerate or delay investment if the firm can increase the total capacity. On the one hand, the firm can increase its profit via a larger production capacity and therefore wants to invest earlier; on the other hand, a higher capacity investment increases the total investment cost. The firm thus waits longer before investing, because a higher demand level is necessary to profit from the investment. If $\alpha = L, \gamma = H$, the cost effect dominates.

I now consider the investment decision of a firm producing two highly substitutable goods. Because of the high substitutability, the firm tends to concentrate on the production of the more profitable A. It strives to avoid competing with itself in the market. Therefore, it is optimal for the firm to switch to flexible capacity early to change the unfavorable output mix imposed by the dedicated capacities. If one of the products is significantly more profitable than the other, the firm is eager to adjust the output mix. Since the market is not large, the firm prefers in the first run to optimally adjust the output rates and not to increase capacity. Therefore, the investment threshold increases with the capacity K_F invested in. The investment

Table 5: Optimal investment threshold ($\theta^*(K_F)$) for adoption of flexible technology for firm currently using dedicated capacities $K_{D,A} = 8$ and $K_{D,B} = 6$. (Parameter values: $L = 0.1$, $H = 0.9$, $r = 0.1$, $\mu = 0.02$, and $c_F = 10$.)

σ	$\theta^*(14)$	$q_A(\theta^*)$	$q_B(\theta^*)$	$\theta^*(15)$	$q_A(\theta^*)$	$q_B(\theta^*)$	$\theta^*(16)$	$q_A(\theta^*)$	$q_B(\theta^*)$
$\alpha = H$, $\gamma = L$									
0.05	143.3	10.98	3.02	34.26	8.45	6.55	27.05	8.75	7.25
0.1	156.3	11.34	2.66	37.76	8.55	6.45	29.84	8.83	7.17
0.15	170.3	11.73	2.27	42.04	8.67	6.33	33.25	8.92	7.08
0.2	184.5	12.12	1.88	46.89	8.80	6.20	37.14	9.03	6.97
$\alpha = L$, $\gamma = L$									
0.05	15.92	10.98	3.02	18.63	12.16	2.84	20.61	13.15	2.85
0.1	17.36	11.34	2.66	20.36	12.59	2.41	22.56	13.64	2.36
0.15	18.93	11.73	2.27	22.29	13.07	1.93	24.75	14.19	1.81
0.2	20.50	12.12	1.88	24.29	13.57	1.43	27.05	14.76	1.24
$\alpha = H$, $\gamma = H$									
0.05	41.38	14	0	35.25	15	0	34.63	16	0
0.1	45.63	14	0	38.83	15	0	38.15	16	0
0.15	50.68	14	0	43.14	15	0	42.41	16	0
0.2	56.20	14	0	47.96	15	0	47.21	16	0
$\alpha = L$, $\gamma = H$									
0.05	4.6	14	0	8.81	15	0	12.17	16	0
0.1	5.07	14	0	9.72	15	0	13.43	16	0
0.15	5.63	14	0	10.84	15	0	14.97	16	0
0.2	6.24	14	0	12.1	15	0	16.73	16	0

cost dominates here. However, if the two products are almost equally profitable, the firm has a better market situation and therefore prefers to increase total capacity to increase the output rate for the more profitable A. Thus, the investment threshold decreases as the capacity investment increases.

Furthermore, the optimal output levels at the time of investment do not change with the profitability α. Uncertainty determines the optimal output split at the time of investment, while α determines the timing. I analyze the investment timing considering the optimal investment threshold θ^*. However, the expected time of investment is equal to the expected time for the demand intercept process to reach the investment threshold θ^*. Since the expected time of investment[4] is monotonic in the investment threshold for the parameter values considered, we can analyze the qualitative results for the investment timing taking into account the investment threshold θ^*.

6 Conclusions

This paper considers the timing and capacity choice of a firm facing stochastic demand. Two types of capacity are distinguished. Flexible capacity allows the firm to produce both products with the same production facility, while dedicated capacity restricts the firm to the production of just one product. The firm makes three decisions: investment time, capacity, and type of capacity, i.e., flexible or dedicated. For the timing and capacity decisions I develop implicit solutions that are investigated numerically. I show that for both flexible and dedicated capacity, the firm invests later if demand uncertainty increases. Flexibility pays off especially when uncertainty is high, substitutability is low, and the profit levels of the two products are substantially different. In the flexible case, under high demand the firm produces only the more profitable product; if demand is low the firm produces both products to increase the total demand. In the dedicated case the firm invests in both capacities if the substitutability rate is low and the profitability of both products sufficiently high. Otherwise the firm will install dedicated capacity for the more profitable product.

A firm currently producing with dedicated capacity may consider switching to flexible capacity. I show that, apart from demand volatility, this decision is substantially affected by the specific product combination. The firm makes the switch even under low demand if the profitability of the two products is substantially different. The option of increasing the total capacity accelerates the investment if the demand for both products is high.

Numerous extensions of this model deserve analysis. We could consider analyzing different cost struc-

[4]The time to reach the investment threshold θ^* starting from a level θ_0, denoted T^*, has expected value

$$E[T^*] = \begin{cases} -\frac{1}{\mu-\frac{1}{2}\sigma^2} ln\left(\frac{\theta_0}{\theta^*}\right) & \text{for } \sigma^2 < 2\mu, \\ \infty & \text{for } \sigma^2 \geq 2\mu. \end{cases}$$

tures, asymmetric demand curves, and different demand functions. Another interesting extension would be multiple investments. Proceeding stepwise would add additional flexibility because the firm could respond to uncertainty by choosing the investment timing at each step. A two-factor model with each products' demand intercept following a separate stochastic process would add demand correlation to the problem. This is a challenging idea for future research.

I analyzed a single-firm model. However, the model has applications in the car and food industries, which are not monopolies. Therefore, a natural next step would be to add strategic interactions to the problem.

A Flexible Capacity

The expected project value for flexible capacity is

$$V(\theta) = \begin{cases} A_1\theta^{\beta_1} + a_1\theta^2 + a_2\theta K_F + a_3 K_F^2 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ B_2\theta^{\beta_2} + \frac{\theta K_F}{r-\mu} - \frac{K_F^2}{r} & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases}. \tag{41}$$

To check the second-order condition for the optimal capacities I derive the second-order derivative of the value function w.r.t. capacity K_F:

$$\frac{\partial V^2(\theta, K_F)}{\partial K_F^2} = \begin{cases} \frac{\partial^2 A_1}{\partial K_F^2}\theta^{\beta_1} + 2a_3 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ \frac{\partial^2 B_2}{\partial K_F^2}\theta^{\beta_2} - \frac{2}{r} & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases} \tag{42}$$

with the first-order derivatives of A_1 and B_2 w.r.t. K_F being

$$\begin{aligned} \frac{\partial A_1}{\partial K_F} &= K_F^{1-\beta_1}\frac{(2-\beta_1)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right], \\ \frac{\partial B_2}{\partial K_F} &= K_F^{1-\beta_2}\frac{(2-\beta_2)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right] \end{aligned}$$

and the second-order derivatives of A_1 and B_2 w.r.t. K_F being

$$\begin{aligned} \frac{\partial^2 A_1}{\partial K_F^2} &= K_F^{-\beta_1}\frac{(1-\beta_1)(2-\beta_1)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right], \\ \frac{\partial^2 B_2}{\partial K_F^2} &= K_F^{-\beta_2}\frac{(1-\beta_2)(2-\beta_2)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right]. \end{aligned}$$

Corollary 1 *The variable A_1 is negative and the variable B_2 is positive for the parameter ranges considered.*

Proof of Corollary 1 A_1 and B_2 are given by

$$\begin{aligned} A_1 &= K_F^{2-\beta_1}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right], \\ B_2 &= K_F^{2-\beta_2}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right]. \end{aligned}$$

Since $\beta_2 < 0$ and $\beta_1 > 1$, we can easily see that the left part of the expression for A_1, i.e., $\left[K_F^{2-\beta_1}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\right]$, is negative, as is the left part of the expression for B_2, i.e., $\left[K_F^{2-\beta_2}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\right]$. Define the function $F(\beta) := \left[\frac{(2-\beta)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta)}{(r-\mu)} - \frac{\beta}{2r}\right]$. The right part of A_1 is given by $F(\beta_2)$ and the right part of B_2 is given by $F(\beta_1)$. If $F(.)$ is positive at β_2 it is proved that A_1 is negative for the parameter ranges considered. If $F(.)$ is negative at β_1 it is proved that B_2 is positive for the parameter ranges considered. Rewriting $F(\beta)$ as

$$F(\beta) = \left[\frac{\mu+\sigma^2}{(r-\mu)(r-2\mu-\sigma^2)}\right] - \beta\left[\frac{2\mu^2+(r+\mu)\sigma^2}{2r(r-\mu)(r-2\mu-\sigma^2)}\right]$$

we can easily see that it is linear with a negative slope, crossing the horizontal axis at $\beta_0 = \frac{2r(\mu+\sigma^2)}{2\mu^2+(\mu+r)\sigma^2} > 0$. We know that $\beta_2 < 0$ and therefore $F(\beta_2)$ must be positive. It remains to show that $\beta_0 < \beta_1$ to verify that $F(\beta_1) < 0$. To show this, we evaluate the quadratic equation $Q(\beta)$ (see Eq. (11)) at β_0. The quadratic expression at β_0 is given by

$$Q(\beta_0) = -\frac{(r-\mu)r\sigma^4(2\mu-r+\sigma^2)}{(2\mu^2+(r+\mu)\sigma^2)^2} < 0. \tag{43}$$

The graph $Q(\beta)$ is an upward-pointing parabola that goes to ∞ as β goes to $\pm\infty$ (see Dixit and Pindyck (1994) on the "fundamental quadratic") and crosses the horizontal axis at β_1 and β_2. Since $Q(\beta_0)$ is negative we know that $\beta_0 < \beta_1$ must hold. Therefore, $F(\beta_1) < 0$. □

B Dedicated Capacity

Derivations for Proposition 2:

1. **Two-Capacity Case:** The optimal capacity is derived by maximizing the project value minus the investment cost. Therefore, we should derive

$$\frac{\partial(V(\theta, K_{D,A}) - c_D K_{D,A})}{\partial K_{D,A}} = 0. \tag{44}$$

Solving the value-matching and smooth pasting conditions assuming that the capacity is a function of the demand intercept θ that solves Eq. (44) leads to the optimal investment thresholds. For $\hat{\theta}_A < \theta < \hat{\theta}_B$ this results in the following two thresholds:

$$\theta^*_{1,1} = \frac{c_D(r-\mu)(1-\gamma)}{(1-\alpha\gamma)},$$
$$\theta^*_{1,2} = \frac{\beta_1 c_D(r-\mu)(1+\gamma-2\gamma^2)}{\beta_1(1+\alpha\gamma)-2(1+\gamma^2(\beta_1-1))}.$$

Since $\theta^*_{1,1} = \hat{\theta}_A$, $\theta^*_{1,2}$ is the unique optimal investment threshold for this region.

The threshold equation for $\theta > \hat{\theta}_B$ is given by

$$\left(\frac{\beta_1 - 2}{\beta_1}\right) \frac{\theta^{*2} r(1+\alpha^2 - 2\alpha\gamma)}{4(r-\mu)^2(1-\gamma^2)} - \left(\frac{\beta_1 - 1}{\beta_1}\right) \frac{\theta^* c_D r(1+\alpha)}{2(r-\mu)(1+\gamma)} + \frac{r c_D^2}{2(1+\gamma)} = 0. \quad (45)$$

We can derive the following two solutions of Eq. (45):

$$\begin{aligned}
\theta_1^* &= \frac{c_D(r-\mu)(1+\alpha)(1-\gamma)}{(1-2\alpha\gamma+\alpha^2)}\left(\frac{\beta_1-1}{\beta_1-2}\right) \\
&\quad + \frac{2(r-\mu)^2(1-\gamma^2)}{r(1-2\alpha\gamma+\alpha^2)(\beta_1-2)}\sqrt{\frac{r^2c_D^2(1+\alpha)^2(\beta_1-1)^2}{4(r-\mu)^2(1+\gamma)^2} - \frac{r^2c_D^2(1-2\alpha\gamma+\alpha^2)\beta_1(\beta_1-2)}{2(r-\mu)^2(1-\gamma^2)(1+\gamma)}}, \\
\theta_2^* &= \frac{c_D(r-\mu)(1+\alpha)(1-\gamma)}{(1-2\alpha\gamma+\alpha^2)}\left(\frac{\beta_1-1}{\beta_1-2}\right) \\
&\quad - \frac{2(r-\mu)^2(1-\gamma^2)}{r(1-2\alpha\gamma+\alpha^2)(\beta_1-2)}\sqrt{\frac{r^2c_D^2(1+\alpha)^2(\beta_1-1)^2}{4(r-\mu)^2(1+\gamma)^2} - \frac{r^2c_D^2(1-2\alpha\gamma+\alpha^2)\beta_1(\beta_1-2)}{2(r-\mu)^2(1-\gamma^2)(1+\gamma)}}.
\end{aligned}$$

Since $\hat{\theta}_B > \theta_2^*$, θ_1^* is the investment threshold of this region.

2. **One-Capacity Case:** The optimal capacity is derived by maximizing the project value minus the investment cost. Therefore, we should derive

$$\frac{\partial(V(\theta, K_{D,A}) - c_D K_{D,A})}{\partial K_{D,A}} = 0 \quad (46)$$

resulting in an optimal capacity K_F^* for a given demand intercept θ of

$$K_{D,A}^*(\theta) = \frac{r}{2(r-\mu)}\left[\theta - (r-\mu)c_D\right]. \quad (47)$$

The project value assuming that the capacity is chosen optimally is given by

$$V(\theta, K_{D,A}^*) - c_D K_{D,A}^* = \frac{r}{4(r-\mu)^2}\theta^2 - \frac{r}{2(r-\mu)}c_D\theta + \frac{r}{4}c_D^2.$$

The value of the option is given by

$$F(\theta) = A_1\theta^{\beta_1}. \quad (48)$$

Value matching and smooth pasting the project value with the value of the option results in the following investment threshold:

$$\theta^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)(r-\mu)\left[\frac{K_D}{r} + c_D\right]. \quad (49)$$

For further explanation for the derivation of the investment threshold by value-matching and smooth pasting see Dixit and Pindyck (1994). Combining Eqs. (47) and (49) gives the optimal investment threshold θ_D^* shown in Eq. (33). The optimal capacity at the time of investment is given by

$$K_D^*(\theta^*) = \frac{1}{(\beta_1 - 2)} r c_D. \quad (50)$$

C Expected Present Value

The formula for $E[e^{-rT}]$, where θ follows the geometric Brownian motion presented in Eq. (3), and T is the random first time that the process reaches a fixed level $\bar{\theta}$ starting from the general initial position θ_0, is given by

$$E[e^{-rT}] = \left(\frac{\theta_0}{\bar{\theta}}\right)^{\beta_1}. \tag{51}$$

See Dixit and Pindyck (1994) for further explanation.

D Incentive to Change from Dedicated to Flexible Capacity

To rule out the scenarios where the downside potential of flexibility exceeds the effect of investment cost, we must impose a lower bound on the unit investment cost c_F. The function $[V_F - V_D](\theta)$ is convex in the demand intercept θ. Therefore, we can conclude that there is no downside potential of flexibility if

$$[V_F - V_D](\theta = 0) - c_F K_F < 0, \tag{52}$$

$$a_3 K_F^2 + \frac{K_{D,A}^2 + 2\gamma K_{D,A} K_{D,B} + K_{D,B}^2}{r} - c_F K_F < 0. \tag{53}$$

Since I assume that $K_F \geq K_{D,A} + K_{D,B}$, (52) is satisfied at $K_F = K_{D,A} + K_{D,B}$. This holds for

$$c_F > \frac{(1-\gamma)}{2r} \frac{[K_{D,A} - K_{D,B}]^2}{[K_{D,A} + K_{D,B}]}. \tag{54}$$

The lower bound is $\hat{c}_F := \frac{(1-\gamma)}{2r} \frac{[K_{D,A} - K_{D,B}]^2}{[K_{D,A} + K_{D,B}]}$.

E Proof of Proposition 3

The equations that implicitly determine the investment threshold θ^* are derived by combining the value matching and smooth pasting conditions given in Eqs. (37) and (38).

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