Intergenerational Risk Sharing and Long-Run Labor Income Risk*

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Abstract

The inability of future generations to share risk with current ones causes financial markets to be incomplete en thus inefficient. By using its financial reserves efficiently, a pension fund is able to transfer current equity risk to future generations, thereby alleviating the "biological" trading constraint that is faced in financial markets. This paper examines how comovements in stock and labor markets affect the gains from intergenerational risk sharing. If stock and labor markets move together in the long run, the human wealth of unborn generations becomes highly correlated with stock returns, which reduces their risk appetite. I show that shifting risk into the future is not optimal anymore once the long-run dynamics of labor income are taken into account. The risk bearing capacity of a pension fund is dramatically decreased if it is unattractive for risk to be transferred to future generations. The results in this paper provide an economic rationale for a tight solvency regime, that requires pension funds to recover from their losses in a short time-period.

Keywords: intergenerational risk-sharing, labor income risk, social security

JEL classification:

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1 Introduction

1.1 General introduction on risk sharing

The Arrow-Debreu theory of general equilibrium teaches that the allocation of risk in financial markets will be Pareto-efficient under certain conditions. In particular, it is required that financial markets are complete. This dissertation is concerned with a deviation from Arrow-Debreu theory arising from the fact that not everyone is born at the same time. Current generations are unable to trade with the unborn generations, which causes financial markets to be incomplete and thus inefficient. This point was made by Diamond (1977), Merton (1983) and Gordon and Varian (1988). More recent contributions include Shiller (2003), Bohn (2006), Smetters (2006), Cui, de Jong, and Ponds (2007), Ball and Mankiw (2007), Gollier (2008) and Gottardi and Kubler (2008). There is thus a role for a social planner to reallocate risk across cohorts.

Risk does not have a negative interpretation here, but is defined as chances of outcomes above or below expectations. Financial-market risk is compensated by gains in expectation, commonly referred to as the risk premium. The existence of a risk premium in financial markets creates an attractive trade-off between risk and return for investors. The objective of a social planner is therefore not to minimize risks, but to allocate risks to those best able to bear them. If designed properly, intergenerational risk-sharing contracts lead to a Pareto-improvement for all generations from an ex-ante perspective. Ex-post realizations, however, may be disadvantageous for some generations. A feasible risk-sharing solution therefore requires participation to be mandatory and can only be enforced under the government's mandate. A private insurance company is unable to commit future generations to join a risk-sharing contract if this is not in their interest. The government has a unique power of taxation which enables it to make commitments on behalf of future generations.

This paper examines risk sharing between non-overlapping generations in the context of a pre-funded social security scheme. Examples include the Social Security Trust Funds in the United States, the Japan Government Pension Investment Fund, the Canada Pension Plan and the ATP fund in Denmark. Most of these funds are diversified with respect to asset class as well as internationally. Some funds, such as the US Social Security trust fund, have been put in place as a buffer against demographic shocks and are expected to deplete in the coming decades. Others, such as the Canada Pension Plan, are permanent in nature and are expected to grow in size in the coming decades. By using its financial buffer efficiently, a pension fund is able to let future generations share in current financial-market risk. In contrast, risk sharing between non-overlapping generations is not possible in the situation in which individuals save and invest on an individual retirement account. Assets are privately owned in a system with individual accounts, which implies that risk sharing is limited to financial-market possibilities. Several countries have established a funded social security tier with individual accounts, for example Australia, Ireland and Estonia. By taking advantage of intergenerational risk sharing, a pension system with publicly-owned assets can outperform a system with individual accounts. Previous studies, e.g. Gollier (2008), have reported large welfare gains associate with risk sharing.

The gains from risk sharing are also eroded by long-run labor income risk. In the long run, stock and labor markets are likely to move together, mirroring changes in the broader economy. If current financial losses (gains) from risk-taking coincide with a decrease (increase) in the expected future wage levels, then future generations are already exposed to current risk via their human wealth. In the presence of long-run labor income risk, it may therefore not be attractive for future generations to share in current risk via a risk-sharing contract.

Many papers have studied risk-sharing in the context of a pay-as-you-go financed pension scheme, see e.g. Bohn (1998), Krueger and Kubler (2002) and Gottardi and Kubler (2008). A pay-as-you-go financed pension scheme is also able to facilitate risk sharing between generations. However, contrary to a funded pension scheme, a pay-as-you-go financed scheme does not involve investments in the financial market.[1] [2] Risk sharing between non-overlapping generations can also take place via wealth transfers within families. Older cohorts leaving intentional bequests to their children can help to share risk between family-members of different cohorts. The size of bequests, however, is rather small for many families. In addition, it is not possible to leave negative bequests, which constrains risk sharing possibilities. This dissertation abstracts from intra-family transfers altogether.

In addition, the analysis is restricted to a single imperfection of financial markets that is addressed by the public pension fund: the inability of non-overlapping generations to trade with each other. The analysis abstracts from other imperfections that can be addressed by pension schemes. For example, pension funds are able to overcome the problem of adverse selection in insurance markets. In addition, pension funds are able to provide insurance

[1]Smetters (2006) points out that an appropriate chosen tax on capital is able to substitute for public investments in the equity market. Hence, the absence of a pre-funded pension scheme does not necessarily imply that there are less possibilities for risk sharing.

[2]Proposals to invest government funds in private securities can be controversial, as illustrated by the debates during the Clinton-administration, see e.g. White (1996), ACSS (1997), GAO (1998) and Greenspan (1999). Public investments in capital markets implies that the government effectively nationalizes a part of the economy. This can be problematic from a governance point-of-view. The decisions of the state as a shareholder may partly be driven by political interest.

against wage-inflation risk, which may not be available in the financial market, see e.g. Cui, de Jong, and Ponds (2007). Public investments in equity can also improve welfare if entry costs prevent some households from investing in the stock market, see e.g. Abel (2001). At the same time, the analysis also abstracts from important disadvantages of collective pension funds. For example, collective arrangements may not offer tailor-made contracts to their participants, thereby ignoring heterogeneity in preferences or personal circumstances. The analysis in this dissertation should therefore not be regarded as a complete cost-benefit analysis of collective pension funds.

Quantitative models are used to answer the central research questions in this dissertation. Quantitative results are important, because it often needs to be determined whether a mechanism of interest plays a dominant role, or whether it is only of second-order importance. The models in this dissertation take the perspective of a small open economy that is too small to affect world prices, and hence asset prices and labor income dynamics are assumed exogenous. The assumption of exogenous factor prices greatly reduces the complexity of analytical expressions and numerical calculations. In addition, the perspective of a closed economy can be problematic in the context of risk sharing between non-overlapping generations. If goods cannot be stored, then a wealth transfer between non-overlapping generations requires the presence of other countries to lend to or borrow from. Even in the situation in which goods can be stored, the assumption of a closed economy can be restrictive, because it is not possible to store a negative amount of goods.

Throughout, the ex-ante welfare criterion is applied to evaluate the welfare of the economic agents. This welfare criterion builds on the Rawlsian approach to social justice. Rawls's thought experiment envisions a hypothetical original position before birth in which individuals agree upon a social contract behind a "veil of ignorance". In the context of risk sharing, the veil of ignorance concerns time-series uncertainty about the returns on financial assets and human wealth.

1.2 Introduction to the paper

The existing literature on risk sharing has pointed out that it can be attractive for future generations to share in current risk. Risk sharing between non-overlapping generations allows risks to be smoothed over a broader base (i.e. a larger number of generations), implying that risk sharing can lead to a Pareto-improvement. The willingness of future generations to share in current risks, however, depends on the risk characteristics of their human wealth. It is less attractive for future generations to share in current risks if these risks relate positively to the return on their human capital. Hence, the question arises: how attractive is it for future generations to share in current risks? Is there really a role for a social planner to transfer risks into the future? This question is addressed in this chapter.

It is natural to conjecture that a sustained period of high (low) economic growth results in strong (weak) stock and labor market performance over the long run. At long horizons, stock and labor markets are likely to move together, mirroring changes in the broader economy. This chapter examines how comovements in stock and labor markets affect the gains from intergenerational risk-sharing, in a rich modeling environment for stock and labor markets. The economic modeling environment is adopted from Benzoni, Collin-Dufresne, and Goldstein (2007), in which labor earnings are cointegrated with dividends on the stock portfolio. The cointegration-framework allows contemporaneous correlations labor income shocks and stock returns to be low or zero (consistent with empirical findings) whereas long-run correlations can be high. Accounting for horizon-dependent correlations is crucial in an evaluation of risk sharing. Different generations face different investment horizons and are thus affected by comovements between stock and labor markets in different ways. Most of the existing studies on risk sharing relies on more stylized two-overlapping-generations (2-OLG) models, which do not allow for time-variation in correlations. An exception is the model of van Hemert (2005). However, in this study, the labor income process is assumed to be stationary, implying that labor income is not risky in the long run.

This chapter contributes to the existing literature in three ways. First, it is shown that the gains from risk sharing can be dramatically reduced once the long-run dynamics of labor income are recognized. < Final results here. >

Second, I find that risk sharing *reduces* the demand for risk assets in comparison to autarky, and hence *increase* the risk premium. This finding contrasts with earlier studies, for example by Gollier (2008) and Beetsma and Bovenberg (2009), who find the opposite result. The reduction in the demand for the risky assets is the result of a negative demand for stocks by unborn generations. It is optimal for young and future generations to be negatively exposed to stock market risk, as this provides a (partial) hedge against shocks to their future labor earnings.

Third, this chapter contributes to the literature by generalizing the economic modeling environment in Benzoni, Collin-Dufresne, and Goldstein (2007) to the case where stock returns are affected by risk sources other than dividend shocks. The extended model is better able to capture the interrelation between stock and labor markets. In particular, the modeling framework allows the volatility of stock returns to exceed the volatility of dividends, consistent with the data. I show that allowing for sources in stock returns variation other than dividend shocks dramatically reduces the effect of cointegration on portfolio holdings. I find that the hump-shape in stock allocations over the life-cycle, reported in Benzoni, Collin-Dufresne, and Goldstein (2007), is *not* robust with respect to alternative parameter choices.

Many studies impose long-run correlations between aggregate labor income and stock returns to be low or zero.[3] This restriction is controversial. In section 2.4 of the introduction, it has been argued that it makes economic sense to assume that the factor shares of labor and capital are stationary, implying that stock and labor markets move together in the long-run. The assumption of stationary factor shares is consistent with the data: although factor shares vary over time, they show no tendency to converge to zero or one. Indeed, Benzoni, Collin-Dufresne, and Goldstein (2007) provide empirical evidence that labor income and dividends are co-integrated.

Many other recent papers have assumed that labor income and dividend flows are cointegrated, see e.g. Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Alternative long-run specifications for the interrelation between stock returns and aggregate labor income have been examined in Campbell (1996), Storesletten, Telmer, and Yaron (2004), Storesletten, Telmer, and Yaron (2007) and Lynch and Tan (2008).[4] Campbell (1996) obtains a high correlation between human capital and market returns in a model where the same (highly time-varying) discount factor is used to discount both labor income and dividends. In his model, labor income follows an AR(1) process and has low contemporaneous correlation with stock dividends. However, the correlation between human capital and market returns is high due to the common and highly varying discount factor.

Bohn (2009) (intergenerational risk sharing and fiscal policy, mimeo) uses a VAR model to estimate 30-year correlations between productivity and capital returns. He reports a positive correlation between 30% and 60%, depending on the specification of the VAR and the cointegrating vector. In addition, the residual volatility in capital returns, conditional on productivity is only a bit higher than that of productivity itself, after correcting for a growth trend

The paper closest related to this chapter is Bohn (2009), who also examines risk-sharing in a setting in which stock and labor markets are subject to a common risk factor. Bohn (2009) finds that efficient risk sharing policies should shift risk away from workers to retirees.

[3]See e.g. Lucas and Zeldes (2006), Jagannathan and Kocherlakota (1996), Sundaresanz and Zapatero (1997), Carroll and Samwick (1997), Gourinchas and Parker (2002), Campbell, Cocco, Gomes, and Maenhout (2001), Cocco, Gomes, and Maenhout (2005), Davis and Willen (2000), Gomes and Michaelides (2005), Haliassos and Michaelides (2003), Viceira (2001).

[4]Earlier studies that investigate a link between aggregate labor income and asset prices include Mayers (1974), Fama and Schwert (1977), Black (1995) and Jagannathan and Kocherlakota (1996).

Via their human wealth, workers bear systematically more risk than retirees. Public pension provisions around the world, which provide relatively safe transfers to retirees, are therefore inefficient in his view. Too much risk is shifted into the future. Bohn (2009) argues that safe pensions can be rationalized as efficient only if preferences display age-increasing risk aversion, such as habit formation.

2 Example: stylized two-agent setting

It is informative to start with a stylized modeling framework with two agents and two periods. The framework extends the two-agent model in Gollier (2008), which abstracts from labor-market issues. Section 2.1 introduces the two-agent model, which features an unborn and a currently-living agent who live in non-overlapping time-periods.

2.1 Model

The model features two agents, where first-born agent $i = 1$ is alive during period 1 and the second-born agent $i = 2$ is alive during period 2. The periods 1 and 2 are non-overlapping, so that it is not possible for the two agents to share risks via a financial market. A long-lived social planner facilitates risk-sharing transfers between the two agents. Risk sharing makes it possible for agent 2 to share in risk that materializes in period 1. Notice, however, that it is not possible for agent 1 to share in risk that realizes in period 2 since the realization occurs after agent 1 has passed away.

Throughout this dissertation, labor earnings and asset returns are assumed exogenous, consistent with the perspective of a small open economy that is too small to affect world prices. Initially, the labor earnings L_i of both agents i (i being equal to 1 or 2) are riskless: $L_i \equiv \bar{L}_i$, where $\bar{L}_i$ is a scalar. The assumption of riskless labor earnings is relaxed in section 2.4, where labor income risk is introduced. Initially, there is only a single source of risk in the model: stock-market risk. Given that only the stock-market risk that materializes in period 1 can be shared between the two agents, I abstract from stock investments in the second period.[5] In the first period, the financial market offers two investment opportunities: a riskless asset with zero return and a risky asset. The net return $\tilde{x}$ on the risky asset is a random variable with mean μ and variance σ^2. The consumption level C_1 of agent 1 consists of labor earnings plus the return on investments minus the risk-transfer from agent 1 to agent

[5] This assumption is harmless when risks are small. However, risk taking in period 2 will decrease the willingness of agent 2 to share in the risks that materialize in the first period if risk exposures are high.

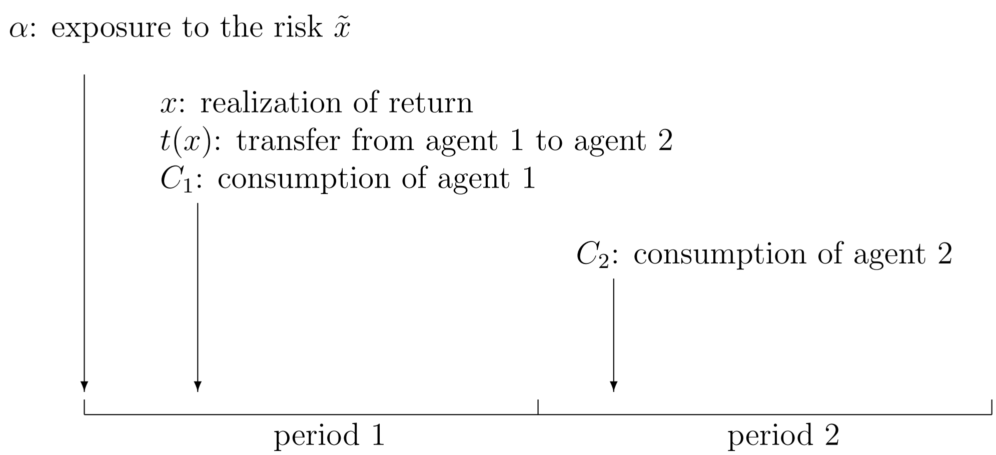


Figure 1: *Time-schedule of section 2.1.*

2, while the consumption level C_2 of agent 2 equals labor earnings plus the risk transfer:

$$C_1 = \bar{L}_1 + \alpha\tilde{x} - t(\tilde{x}), \tag{2.1a}$$

$$C_2 = \bar{L}_2 + t(\tilde{x}), \tag{2.1b}$$

where α denotes the absolute amount invested in the risky asset in period 1 and where $t(\tilde{x})$ is the transfer from agent 1 to agent 2. In an open economy, the intergenerational transfer $t(\tilde{x})$ can be accomplished by lending to or borrowing from abroad. Due to the assumption of a zero risk-free rate, the risk transfer does not accumulate interest between period 1 and 2. Short-selling the risky asset (i.e. $\alpha \geq 0$) does not need to be restricted: the demand for the risky asset is positive as long as the equity premium is positive (i.e. if $\mu > 0$).

Figure 1 shows the time schedule for the two-agent model. The risk exposure α is determined by the social planner before the realization of the return on the risky asset occurs. Subsequently, the realization of the return determines the size of the risk-sharing transfer and the consumption levels of the agents. The risk exposure α cannot be conditioned on the realization of the return on the risky asset, which has not been realized yet at the beginning of the first period.

The two agents have identical preferences given by expected utility over consumption C_i:

$$U_i = \mathbf{E}\left[u(C_i)\right]. \tag{2.2}$$

$\bar{L}_1$	1	labor earnings of agent 1
$\bar{L}_2$	1	labor earnings of agent 2
γ	5	coefficient of relative risk aversion
μ	$30\times0.03=0.9$	expected excess return on stocks
σ	$\sqrt{30}\times0.2=1.1$	volatility of excess return on stocks

Table 1: *Default parameter values in the two-agent model.*

The felicity function features constant relative risk aversion

$$u(C_i) = \frac{C_i^{1-\gamma}}{1-\gamma}, \tag{2.3}$$

where γ denotes the coefficient of relative risk aversion with respect to consumption. The benchmark parameters used in chapter are contained in Table 1. Due to the assumption of a zero riskfree interest rate, $\bar{L}_2$ can be interpreted as the labor earnings of agent 2 discounted back to period 1. For the default parameters, the present discounted value of labor earnings of the two agents is equal. The intuition for the parameter choices for μ and σ is the following. In the situation where stock returns are independent and identically distributed (i.i.d) with a lognormal distribution, the excess mean return over an n-year period approximately equals n times the excess mean return over a 1 year period and the excess volatility over a n-year period approximately equals $\sqrt{n}$ times the excess volatility over a 1 year period. Taking the perspective of a 30-year duration of investments, and choosing the one-year expected excess return and excess volatility equal to 3% and 20% respectively, it follows that their 30-year counterparts are given by $30 \times 0.03 = 0.9$ and $\sqrt{30} \times 0.2 = 1.1$ respectively.

2.2 Autarky

The autarky situation corresponds the case in which there is no transfer between the two agents, i.e. $t(\tilde{x}) = 0$. The autarky solution is well-known and is repeated here for the sake of completeness. In autarky, agent 2 is not exposed to financial-market risk, i.e. $C_2 = \bar{L}_2$. Agent 1 consumes labor earnings $\bar{L}_1$ plus the proceeds from investments in the financial market. The optimal exposure α to the risk $\tilde{x}$ solves from

$$U_1 = \max_{\alpha} \left\{ \mathbf{E}\left[\frac{C_1^{1-\gamma}}{1-\gamma}\right]\right\} = \max_{\alpha} \left\{ \mathbf{E}\left[\frac{\left(\bar{L}_1 + \alpha\tilde{x}\right)^{1-\gamma}}{1-\gamma}\right]\right\}. \tag{2.4}$$

Under the assumption that the portfolio risk is small, the Arrow-Pratt approximation can be applied:

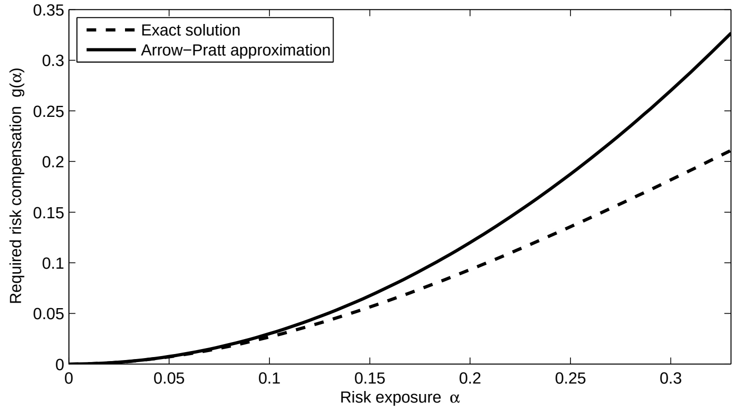


Figure 2: *The exact solution (dashed line) and the Arrow-Pratt approximation (solid line) for the required compensation $g(\alpha)$ for risk as a function of the risk exposure α. The calculations are based upon the default parameters contained in Table 1. The return $\tilde{x}$ on stocks is assumed to adopt a Bernoulli distribution with outcomes -0.2 and +2.0, which yields a mean of 0.9 and a volatility of 1.1, consistent with the default parameters. The exact solution $g(\alpha)$ is the unique solution of the equation $\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\tilde{x}\right)^{1-\gamma}\right]-\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu-g(\alpha)\right)^{1-\gamma}=0$. The Arrow-Pratt approximation is given by equation (2.5): $g(\alpha)\approx\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2$.*

$$\mathbf{E}\left[\frac{\left(\bar{L}_1+\alpha\tilde{x}\right)^{1-\gamma}}{1-\gamma}\right]\approx\frac{\left(\bar{L}_1+\alpha\mu-\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2\right)^{1-\gamma}}{1-\gamma}\equiv\frac{(CEQ_1)^{1-\gamma}}{1-\gamma}, \qquad (2.5)$$

in which CEQ_1 denotes the certainty-equivalent consumption level of agent 1, defined as the non-stochastic consumption level that yields U_1. The Arrow-Pratt approximation is based upon the first two moments (the mean and variance) of the return distribution. Samuelson (1970) provides a discussion on the limitations of mean-variance-analysis in the context of portfolio problems. In equation (2.5), the term $\alpha\mu$ represents the expected return on investments. The term $\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2$ represents the compensation for risk required by agent 1: the agent is indifferent between paying the risk compensation on the one hand and having an exposure α to a pure risk $\tilde{x}-\mu$ on the other hand. Figure 2 illustrates that the Arrow-Pratt approximation is relatively accurate if the risk exposure is small, but becomes less accurate as the portfolio risk increases. The first-order derivative of equation (2.5) solves the optimal risk exposure α:

$$\alpha^{aut}=\frac{\mu}{\gamma\sigma^2}\bar{L}_1. \qquad (2.6)$$

The agent has an appetite for a positive exposure to equity risk as long as the risk premium is positive ($\mu>0$) and the agent is not infinitely risk averse ($\gamma<\infty$). If the risk aversion of the agent goes to zero ($\gamma \to 0$), the agent cares only about the expected return so that the optimal risk exposure goes to infinity if $\mu > 0$. For the default parameters, the agent invests $\alpha^{aut}/\bar{L}_1 = 0.9/(5 \times 1.1^2)$=15% of wealth in the risky asset. The remaining 85% is invested in the riskfree rate.

The welfare gain that results from stock-market participation can be expressed in terms of the percentage change in the certainty-equivalent consumption level of agent 1. Substitution of equation (2.6) into equation (2.5) gives that the welfare gain from risk taking is given by:

$$\%\Delta CEQ_1 = \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2} \text{ x100\%.} \tag{2.7}$$

Note that the expected return from risk taking is $\frac{\mu^2}{\gamma\sigma^2}\bar{L}_1$. Half of that higher expected return is offset by the cost of the attained risk. For the benchmark parameters, risk taking leads to an increase in agent 1's certainty-equivalent consumption level of $0.5 \times 0.9^2/(5 \times 1.1^2)$=6.8%. From this simple exercise it is inferred that the welfare gains from risk taking are large for an individual in autarky.

2.3 Risk sharing

This risk-sharing solution has been treated in Gollier (2008) and is briefly summarized below for the sake of completeness. The objective function of the social planner is to optimize the sum of the certainty-equivalent consumption levels of the two agents. Following Gollier (2008), the risk transfer from agent 1 to agent 2 is characterized by a linear function $t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}$, where α represents the exposure to the risk $\tilde{x}$ in period 1. It follows from equations (2.1) and (2.5) that the certainty-equivalent consumption levels of agents 1 and 2 are given by:

$$CEQ_1(\alpha,\eta) = \bar{L}_1 + (1-\eta)\alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - t_0, \tag{2.8a}$$

and

$$CEQ_2(\alpha,\eta) = \bar{L}_2 + \eta\alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\eta^2\alpha^2\sigma^2 + t_0. \tag{2.8b}$$

The objective function of the social planner is to maximize the certainty equivalent consumption level of the two agents together:

$$\max_{\alpha,\eta}\{CEQ_1 + CEQ_2\} = \max_{\alpha,\eta}\left\{\alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\eta^2\alpha^2\sigma^2\right\}. \tag{2.9}$$

The deterministic transfer t_0 is irrelevant for the optimization problem: a deterministic transfer between agents does not affect the objective function of the social planner. This implies that t_0 can be chosen in such a way that the risk sharing solution is Pareto-efficient, i.e no agent becomes worse off from risk sharing in ex-ante terms. The interval of t_0 for which risk sharing is Pareto-efficient will be derived later in this section. The optimal decisions η^* and α^* are obtained from the first-order derivatives of equation (2.9). Equity risk is allocated according to the relative wealth levels of the two agents:[6]

$$\eta^* = \frac{\bar{L}_2}{\bar{L}_1 + \bar{L}_2}. \tag{2.10}$$

It follows from equation (2.10) that the consumption of both agents is equally elastic to financial shocks:

$$\frac{\partial C_1/C_1}{\partial \tilde{x}} = \frac{\partial C_2/C_2}{\partial \tilde{x}} \approx \frac{\mu}{\gamma\sigma^2}. \tag{2.11}$$

The result in equation (2.11) is referred to as consumption smoothing: financial shocks are smoothed proportionally equally across both periods. The optimal risk exposure solves as

$$\alpha^* = \frac{\mu}{\gamma\sigma^2}(\bar{L}_1 + \bar{L}_2) = \alpha^{aut}\left(\frac{\bar{L}_1 + \bar{L}_2}{\bar{L}_1}\right). \tag{2.12}$$

As pointed out by Gollier (2008), risk sharing increases the demand for the transferrable risk $\tilde{x}$ by a factor $\frac{\bar{L}_1+\bar{L}_2}{\bar{L}_1}$ in comparison to autarky. The intuition for this result is that risk can be spread over a broader base, as risk sharing makes it possible for risk to be shifted towards the future, i.e. to agent 2. For the benchmark parameters, the two agents have equal human wealth in discounted terms, so that it follows from equation (2.12) that the demand for the risky asset doubles.

Note that the risk premium is unaffected by the demand for the risky asset in this partial equilibrium setting. In a small open economy, the increase in the demand for stocks does not affect the global price of risk. In a closed economy, however, an increase in the demand for the risky asset leads to a decrease in the risk premium, thereby reducing agent 1's demand for the risky asset.

The welfare gain from risk sharing is expressed as a fraction of the wealth of the unborn

[6] Notice that the coefficient of relative risk aversion is assumed the same for both agents. If a distinction is made between γ_1 and γ_2 for agent 1 and 2 respectively, then η^* depends on the coefficients of relative risk aversion as well: $\eta^* = \frac{\gamma_1\bar{L}_2}{\gamma_2\bar{L}_1+\gamma_1\bar{L}_2}$. The share η^* of the risk allocated to agent 2 is then an increasing function of the coefficient of relative risk aversion of agent 1: $\partial\eta^*/\partial\gamma_1 > 0$.

agent:

$$\frac{\Delta\left(CEQ_1+CEQ_2\right)}{\bar{L}_2}=\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\text{ x100\%.} \tag{2.13}$$

For the benchmark parameters, risk sharing results in a welfare gain of $(0.5\times 0.9^2/(5\times 1.1^2))$=6.7%. From this simple exercise it is inferred that the gains from risk sharing are potentially large.

The exposure of agent 1 to the risk $\tilde{x}$ remains unchanged in comparison to autarky: agent 1 only takes a fraction $1-\eta=\frac{\bar{L}_1}{\bar{L}_1+\bar{L}_2}$ of the total exposure which increases by a factor $\frac{\bar{L}_1+\bar{L}_2}{\bar{L}_1}$. Thus, agent 1 remains unaffected in comparison to autarky if t_0 is set equal to zero. In this situation, the gain from risk sharing fully accrues to agent 2. On the other extreme, the full gain accrues to agent 1 if $t_0=-\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\bar{L}_2$. Risk sharing is Pareto-efficient from an ex-ante perspective as long as:

$$-\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\bar{L}_2\leq t_0\leq 0. \tag{2.14}$$

The parameter t_0 governs the intergenerational fairness of the risk-sharing contract, i.e. it determines how the gain from risk sharing is divided across the two agents. In this dissertation, two criteria for intergenerational fairness are examined. The first criterion, used in Gollier (2008), imposes that all generations to share proportionally equally in the gains from risk sharing. In the context of the two-agent setting, this criterion implies that the gain from risk sharing is divided proportionally equally across the two agents, i.e. $t_0=-\frac{\bar{L}_2}{\bar{L}_1+\bar{L}_2}\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\bar{L}_2$, where $\frac{\bar{L}_2}{\bar{L}_1+\bar{L}_2}$ denotes the relative wealth of agent 2 and where $\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\bar{L}_2$ denotes the gain from risk sharing as derived in (2.13). For the benchmark parameters, this fairness-criterion implies that both agents gain 3.4% in terms of certainty equivalent consumption. The second criterion, put forward by Teulings and de Vries (2006) and Ball and Mankiw (2007), imposes that all generations are treated equal in terms of market value. In the context of the two-agent setting, this criterion implies that the market value of the risk sharing transfer is equal to zero, i.e. $t_0=0$. Hence, fairness in market terms corresponds to the situation in which the current agent invests according to the autarky solution and in which the unborn agent is able to trade in the financial market before birth (i.e. in period 1). For the benchmark parameters, equality in market terms implies that the gain from risk sharing fully accrues to agent 2. That is: agent 2 gains 6.8% in terms of certainty equivalent consumption, while agent 1 gains nothing.

2.4 Long-run labor income risk

Previous sections assumed the wage rate to be riskless. This section explores how the gains from risk sharing are affected by long-run labor income risk. The labor earnings of agent 2 become stochastic and are therefore denoted by $\tilde{L}_2$ instead of $\bar{L}_2$. For simplicity, the labor earnings of agent 1 remain constant at level $\bar{L}_1$.

I take the perspective where the period-2 labor earnings $\tilde{L}_2$ and period-2 dividend levels $\tilde{D}_2$ are subject to a common risk factor $\tilde{x}$:

$$\tilde{L}_2 = \bar{L}_2 \left(1 + k\tilde{x}\right), \tag{2.15}$$

$$\tilde{D}_2 = \bar{D}_2 \left(1 + \tilde{x}\right), \tag{2.16}$$

where $\bar{L}_2$, $\bar{D}_2$ and k are constants and where $\tilde{x}$ is a random variable with mean μ and variance σ^2. The risk factor $\tilde{x}$ materializes during period 1, and can be interpreted as information on period-2 dividends and labor earnings. Let the period-1 stock price be defined as the discounted value of period-2 dividends. Abstracting from time-variation in discount rates, it follows from equation (2.16) that the period-1 stock return is fully driven by period-2 dividend shocks and is given by $\tilde{x}$.[7] [8] Similarly, it follows from equation (2.15) that the period-1 return on the human wealth of agent 2 is equal to $k\tilde{x}$. Parameter k measures the exposure of period-2 labor earnings to period-1 stock return variation. In the special case where $k = 0$, the labor earnings of agent 2 are constant and the model reduces into section 2.3.

Note that the economic shock $\tilde{x}$ affects stock returns directly (in period 1) and labor earnings with a lag (not until period 2). The underlying intuition for this modeling approach is that period-1 stock prices represent the discounted value of period-2 dividends, and are thus subject to period-2 productivity levels. Similarly, the period-1 return on the human wealth (i.e. the discounted value of future labor earnings) of agent 2 is also affected directly in period 1 by the shock $\tilde{x}$. An alternative reason for why labor earnings are slow to respond to economic shocks is that wages are inelastic at short horizons due to wage rigidity. The

[7]With the period-1 stock price defined as the discounted value of period-2 dividends, it holds that the variation in stock returns is driven by two risk factors: shocks to period-2 dividend levels and shocks in the discount rate. This simple exercise abstracts from time-variation in discount rates, and it follows that all the variation in stock returns is driven by dividend shocks. I will return to this issue later in this section.

[8]Period-2 dividends are stochastic and should therefore be discounted by using a stochastic discount factor. This technique will be applied in subsequent chapters. In this exercise, however, I simply use the riskfree rate (which is assumed equal to zero) to discount period-2 dividends back to period 1.

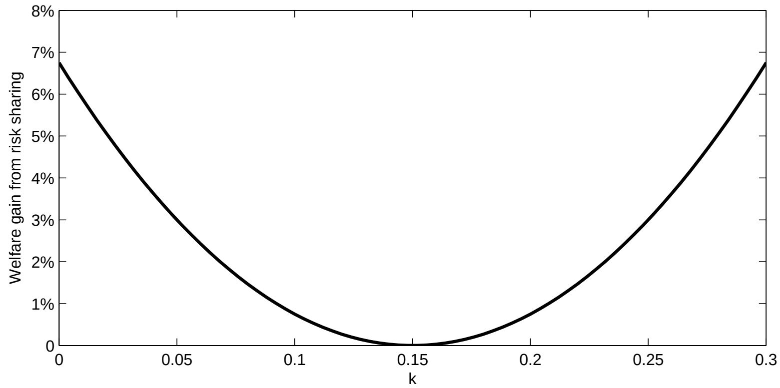


Figure 3: *The welfare gain from risk sharing as a function of* k*. Calculations are based upon the default parameters.*

optimization problem of the social planner, previously given by equation (2.9), alters into:

$$\max_{\alpha,\eta} \left\{ \bar{L}_1 + \bar{L}_2 + \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\left(\eta\alpha + k\bar{L}_2\right)^2\sigma^2 \right\}. \tag{2.17}$$

In equation (2.17), agent 2's exposure to the transferrable risk $\tilde{x}$ equals $\eta\alpha + k\bar{L}_2$, where $k\bar{L}_2$ represents the exposure via human wealth and where $\eta\alpha$ is the exposure through the risk-sharing transfer. The demand for the risky asset solves as:

$$\alpha^* = \frac{\mu}{\gamma\sigma^2}\left(\bar{L}_1 + \bar{L}_2\right) - k\bar{L}_2. \tag{2.18}$$

Comparing equations (2.12) and (2.18), it follows that labor income risk reduces the demand for the risky asset by an amount $k\bar{L}_2$, which is the risk exposure that agent 2 already has to period-1 stock market risk via human wealth. Autarky is Pareto-efficient if

$$k = \mu/(\gamma\sigma^2). \tag{2.19}$$

In this knife-edge case, it holds that agent 2's exposure $k\bar{L}_2$ to the transferable risk $\tilde{x}$ via human wealth equals the optimal exposure $(\mu/(\gamma\sigma^2))\bar{L}_2$. Figure 2.4 illustrates the welfare gain from risk sharing as a function of k for the default parameters. If $k = 0$, labor earnings are riskless and the gain from risk sharing equals 6.7%, as in section 2.3. Autarky is Pareto-efficient if $k = 0.9/(5 \times 1.1^2) = 0.15$, in which case there is no role for the social planner (i.e. $\eta^* = 0$). For values of k smaller than this knife-edge case, the social planner facilitates a positive risk exposure from agent 1 to agent 2 (i.e. $\eta^* > 0$). If k exceeds the knife-edge case, agent 2 wants to be negatively exposed to $\tilde{x}$, i.e. $\eta^* < 0$, as a hedge against future income shocks.

So what is an appropriate choice for the parameter k? Recall from equations (2.15) and (2.16) that k measures comovements between dividends and labor earnings, i.e. between the returns to labor and capital. It makes economic sense to conjecture that the ratio of dividends to labor earnings is constant in the long run, i.e. to assume that dividends and labor earnings are cointegrated. The long-run restriction that the factor shares of labor and capital are stationary is suggested by the form of most production functions used in macroeconomic theory. If labor and capital income are allowed to have independent trends (whether deterministic or stochastic), then the factor share of labor will approach zero asymptotically (if capital income grows faster than labor income) or the factor share of capital will approach zero (in the opposite case). This is contrary to what the data shows: although factor shares vary over time, they show no tendency to converge to zero or one. Indeed, Benzoni, Collin-Dufresne, and Goldstein (2007) provide empirical evidence that dividends and labor earnings are cointegrated. According to their empirical calibration, cointegration takes effect at an horizon of 5-20 years. That is: if dividends double in size over the next 5-20 years, then it can be expected that labor earnings will also approximately double in size over the same period. In our two-agent model, each period has a duration of 30 years, implying that dividends and labor earnings move together at a one-period horizon. From equations (2.15) and (2.16) it follows straightforwardly that the two-agent framework is consistent with the notion of cointegration by setting $k = 1$.

If $k = 1$, the total demand for the risky asset becomes negative, i.e. $\alpha^* < 0$. The negative demand for stocks by agent 2 (to hedge against future shocks in labor earnings) dominates the positive demand for stocks by agent 1. For the benchmark parameters, the total demand for the risky asset becomes negative if $k > 0.3$. A negative demand for the risk asset is feasible in this partial equilibrium framework, which takes the perspective of a small, open economy that is able to trade with foreign countries and is too small to affect the global price of risk. In a closed economy, however, a low risk appetite leads to an increase the equity premium, inducing agent 1 to take more risk. This result stands is sharp contrast to the finding in section 2.3 that risk sharing increases the demand for the risky asset, and hence reduces the equity premium. Although this dissertation focuses on risk sharing, comovements between stock and labor markets thus might have important implications for general equilibrium models that attempt to explain the equity premium puzzle.

The finding that cointegration leads to a negative demand for the risky asset can be interpreted in different ways. One interpretation is that risk sharing does not take place in practice, otherwise the equity premium in financial markets would be higher. Another interpretation is that the model overstates the effect of cointegration on the demand for stocks. There are several reasons for why this may be the case. First, the assumption that the variation in stock returns is fully driven by dividend shocks is inconsistent with the data. The variation in stock returns is subject to risk sources other than dividends shocks. Time-variation in discount rates is an important source of variation in stock-returns that potentially correlates less with human wealth.[9] Also irrationalities, such as mispricing and bubbles in asset prices, may be a source of variation in stock returns. The effect of cointegration on portfolio holdings may also be reduced by foreign stock holdings. The assumption that future labor earnings are cointegrated with dividends makes economic sense in the context of domestic stock holdings, but is less straightforward for foreign holdings. Emerging markets can have growth rates that deviate from those of developed countries over a sustained period of time.

For all the reasons above, the model may overstate the effect of cointegration on portfolio holdings. Therefore, let us introduce an additional source of variation in stock returns to the model. Let the return $\tilde{x}$ on period-1 stock holdings be specified as:

$$\tilde{x} = \mu + \tilde{z}_2 + \tilde{z}_3, \tag{2.20}$$

where $\tilde{z}_2$ and $\tilde{z}_3$ are random variables independent of each other and are distributed with mean zero and variance σ_2^2 and σ_3^2. The term $\tilde{z}_3$ represents the variation in stock returns that is due to shocks in future dividend levels. The term $\tilde{z}_2$ captures all the other sources of variation in stock returns. As explained above, these other sources of risk may include time-variation in discount rates, mispricing, asset bubbles or foreign stock holdings. The total variance of the stock return is denoted by σ^2, and it follows that $\sigma^2 = \sigma_2^2 + \sigma_3^2$. Hence, the extended model allows the volatility of stock returns to exceed the volatility of dividends, consistent with the data. Labor earnings, previously given by equation 2.15, are specified as:

$$\tilde{L}_2 = \bar{L}_2 \left(1 + k\tilde{z}_3\right), \tag{2.21}$$

If $k = 1$, labor earnings are cointegrated with dividends. Consistent with previous sections, the risk-sharing contract is assumed linear and conditioned upon the realization of the risky

[9] Human may may be less affected by time-variation in discount rates due to its short duration. The duration of financial wealth is large: the stock price is based on the discounted value of an infinite flow of future dividends. The duration of human wealth, in contrast, is limited by the retirement date of an investor.

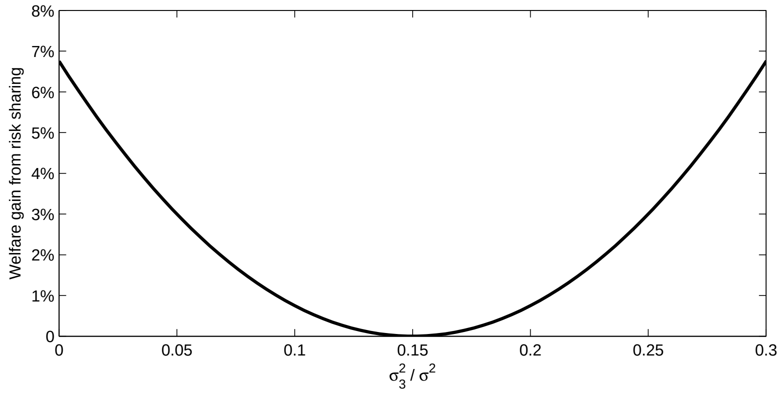


Figure 4: *The gains from risk sharing as a function of the fraction* σ_3^2/σ^2 *of stock return variation due to dividend shocks. Calculations are based upon the default parameters and* $k = 1$*.*

asset return $\tilde{x}$, i.e. $t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}$. The optimization problem becomes:[10]

$$\max_{\alpha,\eta} \left\{ \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\left(\left(\eta\alpha + k\bar{L}_2\right)^2\sigma_3^2 + (\eta\alpha)^2\sigma_2^2\right)\right\}. \tag{2.22}$$

Agent 2's exposure to dividend shocks $\tilde{z}_3$ equals $\eta\alpha + k\bar{L}_2$, where $k\bar{L}_2$ represents the exposure via human wealth and where $\eta\alpha$ represents the exposure via risk sharing. In absence of other sources of variation in stock returns, i.e. $\sigma_2 = 0$, the problem reduces into equation (2.17). The optimal risk exposure α^* solves as:

$$\alpha^* = \frac{\mu}{\gamma\sigma^2}\left(\bar{L}_1 + \bar{L}_2\right) - \frac{\sigma_3^2}{\sigma^2}k\bar{L}_2. \tag{2.23}$$

Labor income risk causes the demand for the risky asset to reduce by an amount $(\sigma_3^2/\sigma^2)k\bar{L}_2$. Assuming $k = 1$, the effect of long-run labor income risk on the demand for the risky asset is determined by σ_3^2/σ^2: the fraction of the variation in stock returns that is due to dividend shocks. If dividend shocks are responsible for a larger fraction of stock return variation, portfolio returns become more strongly correlated with future labor earnings, and agent 2's demand for the risky asset decreases. The demand for the risky asset is positive, i.e. $\alpha^* > 0$, if less than 30% of the variation in stock returns is due to dividend shocks.

The knife-edge case in which autarky is Pareto-efficient is given by $k = \mu/(\gamma\sigma_3^2)$. Let us apply the numerical example parameters to this knife-edge situation. Assuming $k = 1$,

[10] The optimization problem in equation (2.22) is equivalent to that in equation (2.17), with k being replaced by $k\sigma_3^2/\sigma^2$.

the knife-edge case is given by $\sigma_3^2 = \mu/\gamma = 0.9/5 = 0.18$, implying that the gains from risk sharing are fully eroded if $\sigma_3^2/\sigma^2 = 0.18/1.1^2 = 15\%$ of the variation in stock returns is due to dividend shocks. This result is graphically illustrated in Figure 4. In absence of dividend shocks, i.e. $\sigma_3 = 0$, labor earnings and stock returns do not have a common risk factor and the gain from risk sharing equals 6.7% as in section 2.3. In the knife-edge case where σ_3^2/σ^2=15%, autarky is Pareto-efficient and the gains from risk sharing are fully eroded.

It is inferred from this stylized analyses that the gains from risk sharing are large if labor-market aspects are ignored. These gains, however, are reduced once labor-market distortions and the long-run dynamics of labor income are recognized. The highly stylized two-agent model does not give reliable quantitative answers. The remaining chapters therefore extend the analysis to a richer modeling environment.

3 The model

This section presents the model for overlapping generations, the stock market, the labor market and individual preferences. The model for the stock and labor market is an extension of Benzoni, Collin-Dufresne, and Goldstein (2007), in which stock return variation is also driven by sources of risk other than dividend shocks.

Stock return process

The economy consists of two assets: a riskless asset and portfolio of stocks. The riskless asset offers a real instantaneous return r. Let dividends D_t on the stock portfolio be given by Geometric Brownian Motion:

$$\frac{dD_t}{D_t} = g_d dt + \sigma_3 dz_{3,t}, \tag{3.1}$$

where $dz_{3,t}$ is a standard Wiener process and where g_d denotes the growth rate of dividends. Assuming the price of risk to be constant, and defining the stock price as the discounted value of future dividends, Benzoni, Collin-Dufresne, and Goldstein (2007) show that the excess return on the investment strategy X_t that reinvests all proceeds (dividends and capital gains) in the stock market portfolio is given by: $dX_t/X_t = \mu dt + \sigma_3 dz_{3,t}$, in which μ denotes the expected excess return on stock holdings. Notice that, in this specification, all the variation in stock returns is due to dividend shocks. Furthermore, it holds that the volatility of stock returns is equal to the volatility of dividends. As explained in section 2.4, these two model features are unattractive. The volatility of stock returns is observed to be substantially larger than the volatility of dividends. Stock returns are likely to be affected by other risk sources, such as time-variation in discount rates, mispricing and asset bubbles. Therefore, consistent with the analysis in section 2.4, the specification for stock returns in Benzoni, Collin-Dufresne, and Goldstein (2007) is extended as follows:

$$\frac{dX_t}{X_t} = \mu dt + \sigma_2 dz_{2,t} + \sigma_3 dz_{3,t}, \tag{3.2}$$

where $dz_{2,t}$ is a standard Wiener process independent of $dz_{3,t}$ that captures sources of stock return variation other than dividend shocks. The expected excess return on stocks equals μ and the volatility of the stock portfolio equals $\sigma_2^2 + \sigma_3^2 \equiv \sigma^2$.

The labor income process

At each point in time t, all working individuals earn the same labor income level L_t. Hence, the labor income specification does not include a career-patterns over the life-cycle and abstracts from individual-specific and cohort-specific shocks.[11] Also, the model does not include a . I follow Benzoni, Collin-Dufresne, and Goldstein (2007) by assuming that aggregate labor labor income process L_t and the dividend process D_t are cointegrated. Let the variable $y(t)$ denote the logarithmic labor-to capital income ratio:

$$y_t = l_t - d_t - \overline{ld}, \tag{3.3}$$

where $l_t = log[L_t]$ and $d_t \equiv log[D_t]$, and where the constant $\overline{ld}$ is the long-run logarithmic ratio of aggregate labor income to dividends. Cointegration between labor earnings and dividends is ensured by imposing that the (logarithmic) labor-to capital income ratio is a mean-reverting process:

$$dy_t = -\kappa y_t + \nu_1 dz_{1,t} - \nu_3 dz_{3,t}, \tag{3.4}$$

where $z_{1,t}$ is a standard Brownian motion independent from $z_{2,t}$ and $z_{3,t}$. The coefficient κ measures the speed of mean reversion for the process y_t.[12] By applying Ito's lemma to equation (3.1), and substituting equations (3.3) and (3.4), it follows that the (logarithmic)

[11] Individual-specific shocks are ignored because I focus on *inter*-generational transfers. Individuals can insure themselves against individual-specific shocks via *intra*-generational risk-sharing, although this is difficult in practice due to problems related to moral-hazard, adverse-selection and limited liability.

[12] In the presence of cointegration, i.e. if $\kappa > 0$, The term $z_{1,t}$ captures temporary income shocks and has only a minor effect on decision making. Inclusion of the term, however, is important when calibrating the model to the data.

π	0.20	contribution rate
n	40	number of working years
m	20	number of retirement years
γ	5	relative risk aversion
r	0.02	riskfree rate
μ	0.03	expected excess return on stocks
g_D	0.018	expected growth rate of dividends
σ_3	0.1	volatility of dividends
σ	0.2	volatility of stock portfolio
κ	0.1	co-integration coefficient
ν_1	0	
ν_3	σ_3	

Table 2: Default model parameter values.

income process l_t is given by:

$$dl_t = \left\{ -\kappa y_t + g_d - \frac{\sigma_3^2}{2} \right\} dt + \nu_1 dz_{1,t} + (\sigma_3 - \nu_3) dz_{3,t}. \tag{3.5}$$

Since $z_{1,t}$ is orthogonal to $z_{3,t}$, it follows that the contemporaneous correlation between stock returns and labor income shocks is given by:

$$\text{corr}(dlog[X_t], dlog[L_t]) = \frac{\sigma_3 - \nu_3}{\sqrt{v_1^2 + \sigma_2^2 + (\sigma_3 - \nu_3)^2}}. \tag{3.6}$$

Hence, labor income is contemporaneously uncorrelated with stock returns in the special case where $\sigma_3 = \nu_3$. Cointegration, however, causes the correlation between stock returns and labor earnings to be an increasing function of the horizon.

Benzoni and Chyruk (2009) clarify how the co-integration framework relates to the more traditional specifications for labor income risk. They show that, in absence of cointegration, i.e. if $\kappa \to 0$, and in absence of an instantaneous correlation between labor income and stock returns, i.e. if $\sigma_3 = \nu_3$, the specification is nearly identical to a framework with time-invariant correlations as in Cocco, Gomes, and Maenhout (2005). In this situation, stock returns and labor earnings follow independent random walks, and the term $z_{1,t}$ captures permanent income shocks.[13] If, in addition to these conditions, ν_1 is set equal to zero, the model features deterministic labor earnings, which grow at a rate equal to g_D, i.e. $L_v = L_t e^{g_D(v-t)}$ for all $v > t$. Labor earnings become constant by additionally setting $g_D = 0$.

Default model parameters

The default model parameter choices are contained in Table 2. The default choice $g_D = 0.018$ for the expected growth rate of dividends is adopted from Benzoni, Collin-Dufresne, and Goldstein (2007). The benchmark choice for the cointegration coefficient κ is chosen to be 0.1. Benzoni, Collin-Dufresne, and Goldstein (2007) find an estimate for the cointegration coefficient of 0.2052 when using data going back to 1929, while the estimate is as low as 0.0475 when relying on the post-World War II sample period. Therefore, this chapter will provide a sensitivity analysis of results for alternative parameter choices, namely $\kappa = 0.05$ and $\kappa = 0.2$. Notice that the choice for κ does not affect the long-run growth rate of labor earnings. As long as $\kappa > 0$, the model features a stationary dividend-earnings ratio, implying that the long-run growth rate of labor earnings coincides with the long-run growth rate g_D of dividends.

I choose $\sigma_3 = 0.1$ as the default parameter for the volatility of dividends, implying that dividend shocks are responsible for σ_3^2/σ^2=0.25% of the total variation in stock returns. In the sensitivity analysis, results are also shown for the cases where $\sigma_3 = 0.05$ and $\sigma_3 = 0.2$. If $\sigma_3 = 0.2$, all the variation in stock returns is due to dividend shocks, i.e. σ_3^2/σ^2=100%, and the model reduces into Benzoni, Collin-Dufresne, and Goldstein (2007). The parameter ν_3 is determined such that there is a zero contemporaneous correlation between labor income growth and stock market returns, consistent with empirical findings. From equation (3.6) it follows that this is accomplished by setting $\nu_3 = \sigma_3$. This equality is maintained in the sensitivity analysis with respect to σ_3. That is: if σ_3 is adjusted, then also ν_3 is adjusted in order to preserve a zero contemporaneous correlation between labor income growth and stock market returns. I abstract from temporary labor income shocks, i.e. $\nu_1 = 0$. As explained in footnote 12, temporary income shocks have only a minor effect on portfolio holdings.

Example scenarios

Figures 5 and 6 illustrate the effect of a dividend shock on the other model parameters for various parameter choices. The solid line in both figures corresponds to the benchmark. The Figure illustrates that shocks in dividends are permanent in nature, which is due to the assumption that dividends follow a Geometric Brownian motion. A negative shock in dividends leads to an immediate drop in the stock price, and thus has an immediate impact on stock returns. Due to the assumption that $\sigma_3 = \nu_3$, labor earnings are instantaneously

[13] Thus, whereas the labor income shocks $z_{1,t}$ are temporary in nature in the presence of cointegration (see footnote 12), these shocks become permanent in the absence of cointegration.

unaffected and respond to a dividend shock with a lag. It can be shown that $1 - e^{-1}$=63.2% of the total impact on labor earnings materializes within the first $1/\kappa$ years after the shock. As a rule-of-thumb, it can therefore be said that cointegration takes effect in approximately $1/\kappa$=1/0.15=7 years.

Figure 5 provides a sensitivity analysis with respect to the cointegration coefficient κ. If the cointegration coefficient κ is reduced, from 0.15 to 0.05, labor earnings respond slower to a dividend shock. The horizon at which cointegration takes effect increases from 1/0.15=7 to 1/0.05=20 years. The long-run effect of a dividend shock on labor earnings, however, does not depend on the cointegration coefficient. In absence of cointegration, if $\kappa = 0$, labor earnings are unaffected by dividend shocks. Figure 6 provides a sensitivity analysis with respect to the volatility of dividends σ_3. If σ_3 increases from 0.1 to 0.2, the relative importance of dividend shocks in the total variation of stock returns increases from 25% to 100%. As a result, stock returns and labor earnings are much more affected by dividend shocks compared to the benchmark case.

Time-varying correlations

Figure 7 shows the correlation between current stock returns and future labor earnings as a function of the horizon. Labor income and stock returns are contemporaneously uncorrelated, but the correlation is an increasing function of the horizon. Due to temporary labor income shocks $z_{1,t}$, the correlation between stock and labor markets is not perfect in the long-run. Long-run correlations are increasing in the cointegration coefficient κ and the volatility of dividends σ_3.

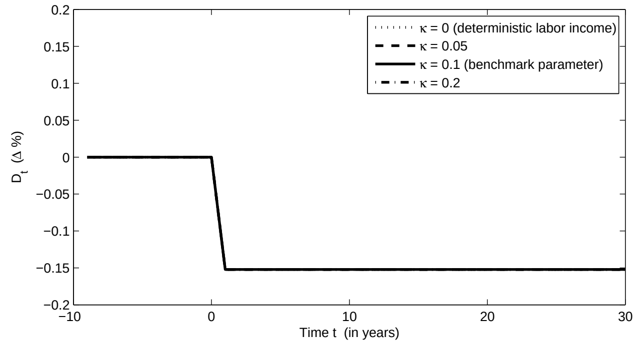
The overlapping generations framework

There are $n + m$ overlapping generations. Each generation participates in the labor market for a period of n years and is subsequently retired for a period of m years. All generations are equal in size and the size of each generation is normalized to unity.

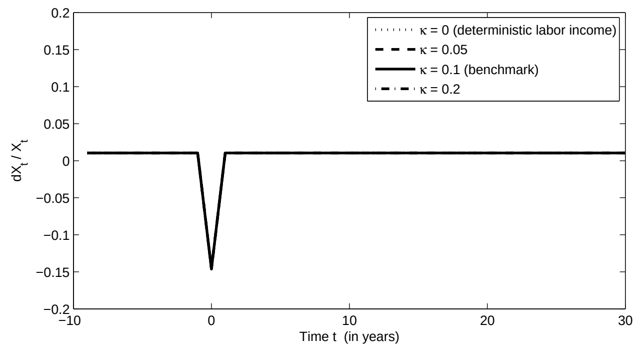
Preferences

The model takes the perspective of defined-contributions. Thus, individuals save a a fixed faction of labor earnings during the working period.[14] Consumption levels during the working

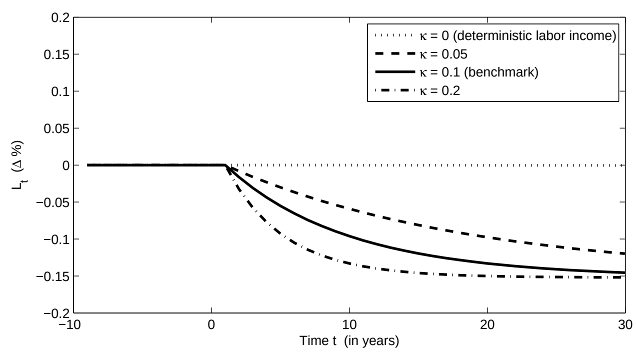
[14]These two assumptions are rather convenient in this chapter because they do not require the social planner to solve n contribution decisions and m benefit payout decisions at each point in time.



(a) Dividend level $D_t(\%\Delta)$

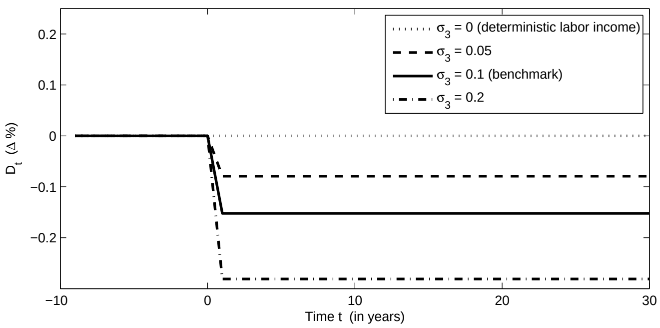


(b) Excess return dX_t/X_t

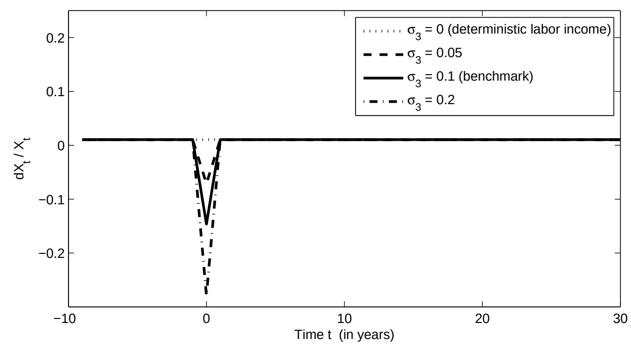


(c) Labor earnings $L_t(\%\Delta)$

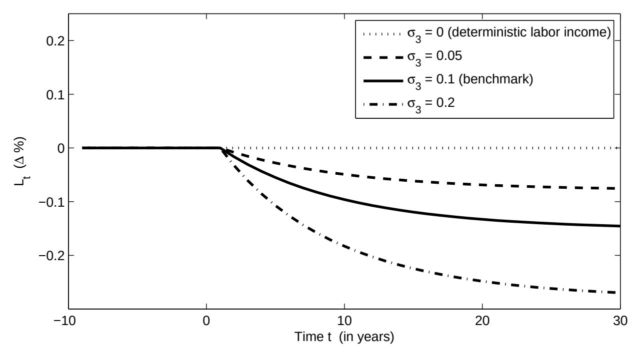
Figure 5: *The effect of a dividend shock $z_{3,v}$ on the dividend level D_t, the excess return dX_t/X_t and labor earnings L_t, for three alternative parameter choices for the cointegration coeffcient κ. Both scenarios corresponds to the scenario in which $z_{1,t} = z_{2,t} = z_{3,t} = 0$ for all t, except for a one-time shock $z_{3,v} = -1.65$ at some point in time v, which is normalized to zero. Calculations are based upon the default parameters as contained in Table 2. Plots are based upon Euler-iterations with a time-step $\Delta t = 1$.*



(a) Dividend level $D_t(\%\Delta)$

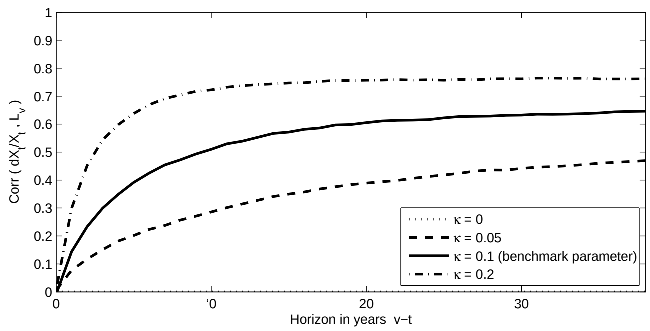


(b) Excess return dX_t/X_t

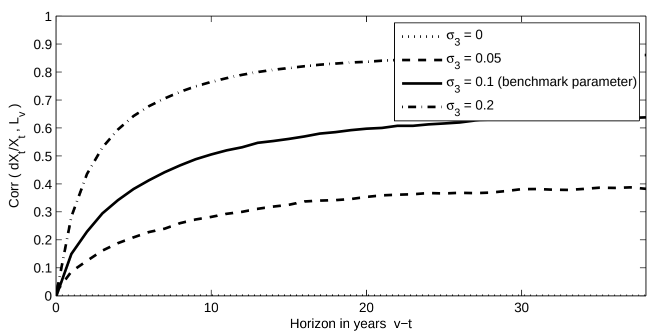


(c) Labor earnings $L_t(\%\Delta)$

Figure 6: *The effect of a dividend shock $z_{3,t}$ on the dividend level D_t, the excess return dX_t/X_t and labor earnings L_t, for three alternative parameter choices for the cointegration coefficient κ. Both scenarios corresponds to the scenario in which $z_{1,t} = z_{2,t} = z_{3,t} = 0$ for all t, except for a one-time shock $z_{3,v} = -1.65$ at some point in time v, which is normalized to zero. Calculations are based upon the default parameters as contained in Table 2. Plots are based upon Euler-iteration with a time-step $\Delta t = 1$.*



(a) Sensitivity to κ



(b) Sensitivity to σ_3

Figure 7: *The correlation between current stock returns dX_t/X_t and future labor earnings L_v as a function of the horizon $v-t$. Calculations are based upon the default parameters as contained in Table 2, except that ν_1 is set equal to 0.05 instead of 0.*

period are thus exogenous. In addition, it is assumed that retirement wealth is converted into a flat annuity with some payoff level b_s during the m-year retirement period. As a result, the preferences over consumption of cohort s can be defined as a function of b_s:

$$U_s = \mathbf{E}_{t_0}\left[\frac{1}{1-\gamma}b_s^{1-\gamma}\right]. \tag{3.7}$$

4 Autarky

The autarky framework corresponds to the setting in which all individuals save and invest on an individual savings account.

4.1 Optimization problem

Individual investors save a fixed fraction π of labor earnings during their working period and use the remaining fraction $1-\pi$ for consumption. Initial financial wealth in the individual savings account is assumed equal to zero for all cohorts:

$$F_{s,s} = 0, \tag{4.1a}$$

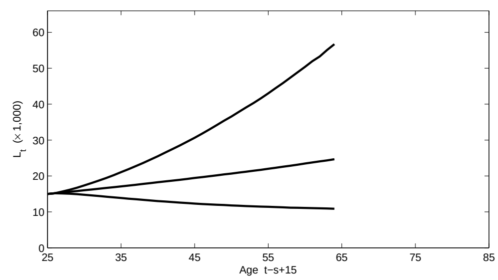
where $F_{s,t}$ denotes the amount of wealth at time t in the savings account of an individual of cohort s. The financial wealth on the retirement savings accounts is subject to the intertemporal budget constraint during the working period:

$$dF_{s,t} = rF_{s,t}dt + \alpha_{s,t}dX_t/X_t + \pi L_t dt, \tag{4.1b}$$

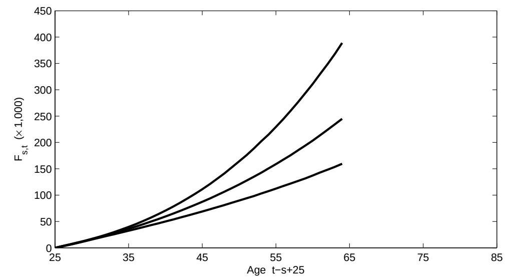
for all $s \leq t \leq s+n$, where $\alpha_{s,t}$ denotes the amount invested in the risky asset by an individual in cohort s at time t. Final wealth at the retirement date $s+n$ is converted into a flat m-year annuity. Assuming annuities to be priced in an actuarially fair way, the terminal wealth condition is given by

$$F_{s,s+n} = \int_0^m e^{-rv}b_s dv = \frac{b_s}{r}\left(1-e^{-rm}\right). \tag{4.1c}$$

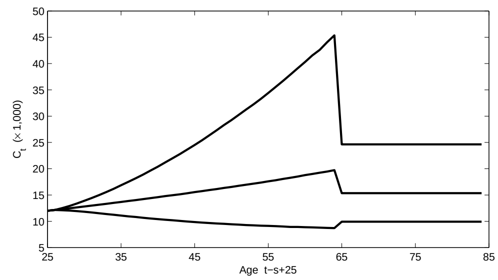
The individual investor in autarky maximizes preferences as specified in 3.7 with respect to portfolio choices $\alpha(\cdot)$, subject to the budget constraints in equation (4.1).



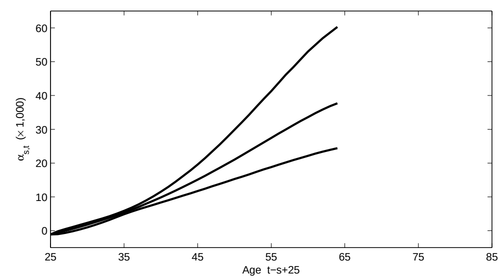
(a) Labor income L_t



(b) Financial wealth $F_{s,t}$



(c) Consumption $C_{s,t}$



(d) Amount $\alpha_{s,t}$ invested in stocks

Figure 8: *5%, 50% and 95% quantiles for a number of model variables in autarky. Calculations are based upon the default parameters as contained in Table 2. Without loss of generality, the initial labor income level L_s is normalized to 15,000.*

4.2 Solution

Except for a few special cases, the model of section 3 cannot be solved analytically. The model is therefore solved numerically by using backward induction, state-space discretization, spline interpolation and Gaussian quadratures. There are three state variables in the model: financial wealth $F_{s,t}$, labor earnings L_t and the (logarithmic) dividend-earnings ratio y_t. As explained in Benzoni, Collin-Dufresne, and Goldstein (2007), the model has a scaling feature which reduces the number of state variables to two: F_t/L_t and y_t.

Figure 8 shows the results for the benchmark parameters. Notice that the confidence intervals for labor income display a diverging pattern over time. Labor income is generally increasing over time, although it can be decreasing over substantial periods of time in some scenarios. Financial wealth is generally increasing over the working life. With 90% probability, the wealth accumulated at the retirement date is higher than 160,000 and lower than 427,000. In striking contrast to most life-cycle models, the demand α for stocks is increasing over the life-cycle. Labor income risk reduces the risk appetite of young investors, because stock returns are positively correlated with future shocks in labor income. At young ages, the demand for stocks is even slightly negative. This negative exposure to stocks provides

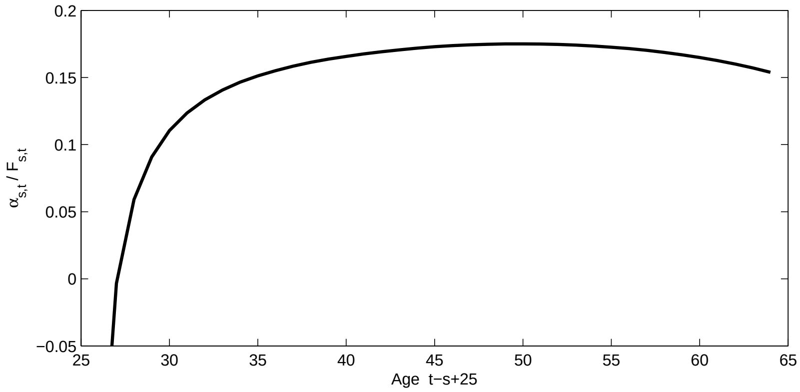


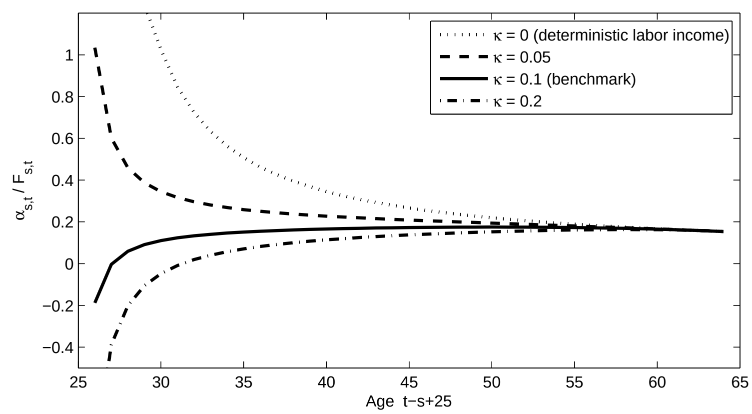
Figure 9: *The 50% quantile for the fraction $\alpha_{s,t}/F_{s,t}$ of financial wealth invested in stocks. Calculations correspond to the benchmark parameters contained in Table 2.*

the young with a (partial) hedge against shocks in future labor earnings.

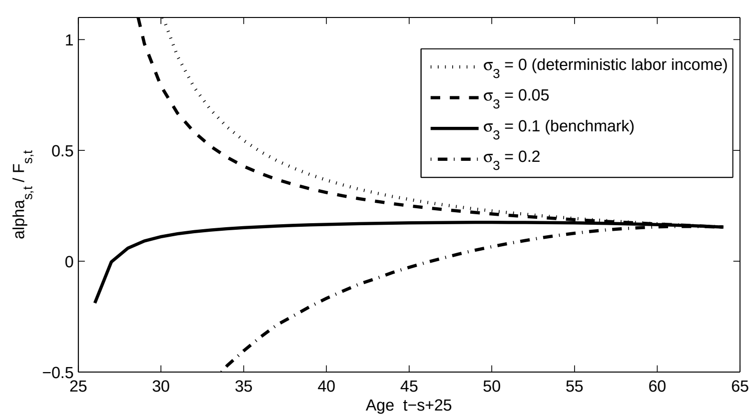
Figure 9 illustrates the fraction $\alpha_{s,t}/F_{s,t}$ of financial assets invested in stocks. The figure illustrates that cointegration causes the fraction of wealth invested in stocks to be increasing during the first-half of the working period. Notice, however, that the effect of cointegration diminishes at ages close to retirement. The duration of the human wealth of workers close to retirement is relatively small. For workers close to retirement, comovements between stock and labor markets at long horizons become irrelevant and hence do not affect portfolio holdings. Therefore, at later ages, the fraction of financial assets becomes decreasing again, similar to the result obtained in absence of labor income risk. For the benchmark parameters, cointegration between stock and labor markets therefore causes portfolio holdings to display a hump-shape over the-life cycle. Benzoni, Collin-Dufresne, and Goldstein (2007) have used this finding to explain low participation levels in the stock market for young individuals (the stock-market participation puzzle).

Figure 10(a) illustrates that the hump-shape in portfolio allocation to stocks over the life-cycle is not robust with respect to changes in the cointegration coefficient. If κ is reduced from 0.15 to 0.05, the hump-shape in portfolio allocation to stocks disappears. Instead, stock allocations become decreasing over the life-cycle. Figure 10(b) illustrates that that the hump-shape is also not robust with respect to changes in the volatility of dividends σ_3. Stock allocations become decreasing over the life-cycle if σ_3 is decreased from 0.10 to 0.05, whereas they become increasing over the life-cycle if σ_3 is increased from 0.1 to 0.2.

This chapter abstracts from the borrowing constraint as well as other constraints on investment choices. For the benchmark parameters, the borrowing constraint is not binding because financial wealth levels remain positive is practically all scenarios, as illustrated in



(a) Sensitivity with respect to κ



(b) Sensitivity with respect to σ_3

Figure 10: *The 50% quantile for the fraction $\alpha_{s,t}/F_{s,t}$ of financial wealth invested in stocks, for three different choices of the cointegration coefficient κ and the volatility of dividends σ_3. Calculations correspond to the benchmark parameters contained in Table 2.*

Figure 8. However, the optimal strategy does require a short position in stocks.

5 Risk sharing

5.1 Optimization problem

Consistent with previous chapters, the social planner takes the form of a benevolent pension fund. All working cohorts pledge a fraction π of their labor earnings to the pension fund, where the savings rate π is the same as in autarky. At the retirement date, participants receive an m-year flat annuity with payoff level b_s. The variable b_s is a decision variable of the pension fund that is determined at the retirement date of cohort s, i.e. at time $s+n$. The intertemporal budget constraint of the pension fund is given by:

$$dF_t = \begin{cases} rF_t dt + \alpha_t dX_t/X_t + n\pi L_t dt & \text{if } t \notin \mathbb{N} \\ rF_t dt + \alpha_t dX_t/X_t + n\pi L_t dt - \frac{b_{t-n}}{r}\left(1-e^{-rm}\right) & \text{if } t \in \mathbb{N} \end{cases} \tag{5.1}$$

where F_t denotes the value of financial assets of the pension fund at time t, where α_t denotes the amount invested in the stock market by the pension fund at time t and where b_{t-n} denotes the payoff-level of the annuity that is received the retiring cohort, i.e. the cohort that entered the labor market at time $t-n$. Benefit payments are only made at discrete points in time, i.e. at times $t \in \mathbb{N}$, when the oldest working cohort retires.

To determine the initial wealth level of the pension fund, I follow the approach of Gollier (2008). That is, we take the perspective of a pension reform in which the n working cohorts in autarky agree to transfer their wealth to a social planner. Before the date of the pension reform, individual investors are saving on individual retirement accounts according to the optimal autarky portfolio rule. The date of the reform is normalized to t_0. The value of pension fund assets F_{t_0} at time t_0 is stochastic, as it depends on the wealth in the individual accounts of the generations that are alive at the time the transition. Also the labor income level L_{t_0} and the earnings-to-dividends ratio y_{t_0} are stochastic. There are thus many possible scenarios for the reform. However, the variables y_{t_0} and F_{t_0}/L_{t_0} both adopt a stationary distribution. This implies that the normalization of the date of the reform to t_0 is without loss of generality. The analysis is restricted to a single scenario for the reform, namely the scenario in which the values of y_{t_0} and F_{t_0}/L_{t_0} are equal to their unconditional means (i.e. their long-term averages).

Participants do not save or investment outside the pension fund. The pension fund optimizes the aggregated utility of all currently-living and future cohorts:

$$\max_{\alpha,b} \left\{ \mathbf{E}_{t_0} \left[\sum_{t=t_0}^{\infty} \delta^{-t} U_s \right] \right\}, \tag{5.2}$$

with respect to the decisions for portfolio holdings and benefit payments, subject to the budget constraint in equation (5.1). Parameter δ represents the discount factor that the social planner uses to weigh the relative importance of the cohorts. Consistent with Gollier (2008), parameter δ is set by the social planner such that the welfare gain from risk sharing is proportionally equally divided among cohorts:[15]

$$\frac{CEQ_{s'}}{CEQ_s} = \frac{CEQ_{s'}^{aut}}{CEQ_s^{aut}}, \tag{5.3}$$

for all cohorts $s, s' > t_0$, where CEQ_s and CEQ_s^{aut} represent the certainty-equivalent retirement consumption level of cohort s in the pension fund and in autarky respectively, and is defined as:

$$CEQ_s^{1-\gamma} 1 - \gamma \equiv \mathbf{E}_{t_0} \left[\frac{b_s^{1-\gamma}}{1-\gamma} \right]. \tag{5.4}$$

The welfare gain from risk sharing can now be expressed in terms of the percentage change in CEQ_s, which is the same for all cohorts. Risk sharing is Pareto-efficient, because the social planner is able to replicate the optimal individual strategy, which ensures that all cohorts are at least as well off as in autarky.

5.2 Solution

To be written.

6 Conclusion

To be written.

[15] In chapters **??** and **??**, there was no growth in labor income, implying that $CEQ_s^{aut} = CEQ_{s'}^{aut}$ for all s,s', which causes equation (5.3) to simplify into equations (**??**) and (**??**).

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