10th Winter School– Energy Markets

Lecture 3
Risk Premia in Energy Markets II

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Agenda

Market Risk Premium – Players

The main motivation for players to engage in forward contracts is that of risk diversification.

Producers have made large investments with the aim of recouping them over a long period of time as well as making a return on them.

Retailers (which might be intermediaries and/or use the commodity in their production process) also have an incentive to hedge their positions in the market by contracting forwards that help diversify their risks.

Market Risk Premium – Qualitative

Exposure to the market will differ both between producers and retailers as well as within their own group.

For example, a large producer will generally be exposed to market uncertainty for a longer period of time, perhaps determined by the remaining life of the assets, whilst retailers will tend to make decisions based on a shorter time scale.

So the need for risk-diversification has a temporal dimension.

Market Risk Premium

- These differences in the desire to hedge positions are employed to explain the market risk premium and its sign.
- Retailers are less incentivized to contract commodity forwards the further out we look into the market.
- In contrast, on the producers' side the need to hedge in the long-term does not fade away as quickly.

Market Risk Premium

- We associate situations where $\pi(t, T) > 0$ with the fact that retailers' desire to cover their positions 'outweighs' those of the producers, resulting in a positive market risk premium.
- The mirror image is therefore one where the producers' desire to hedge their positions outweighs that of the retailers resulting in a negative market risk premium.

Representative Agents

- We describe producers' and retailers' preferences via the utility function of two representative agents.
- Agents must decide how to manage their exposure to the spot and forward markets for every future date T.
- A key question for the producer is how much of his future production, which cannot be predicted with total certainty, will he wish to sell on the forward market or, when the time comes, sell it on the spot market.
- Similarly, the retailer must decide how much of her future needs, which cannot be predicted with full certainty either, will be acquired via the forward markets and how much on the spot.

Representative Agents

We approach this financial decision and equilibrium price formation in two steps.

- First, we determine the forward price that makes the agents indifferent between the forward and spot market.
- Second, we discuss how the relative willingness of producers and retailers to hedge their exposures determines market clearing prices.

Representative Agents

We assume that the risk preferences of the representative agents are expressed in terms of an exponential utility function parameterized by the risk aversion constant $\gamma > 0$;

$$U(x) = 1 - \exp(-\gamma x)\,.$$

We let $\gamma := \gamma_p$ for the producer and $\gamma := \gamma_c$ for the retailer.

The Model

We assume that the electricity spot price follows a mean-reverting multi-factor additive process

$$S_t = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t) \qquad (1)$$

where $\Lambda(t)$ is the deterministic seasonal spot price level, while $X_i(t)$ and $Y_j(t)$ are the solutions to the stochastic differential equations

$$dX_i(t) = -\alpha_i X_i(t)\, dt + \sigma_i(t)\, dB_i(t) \qquad (2)$$

and

$$dY_j(t) = -\beta_j Y_j(t)\, dt + dL_j(t). \qquad (3)$$

$B_i(t)$ are standard independent Brownian motions, $\sigma_i(t)$ are det. vola. functions and $L_j(t)$ are independent Lévy processes.

The Model

The processes $Y_j(t)$ are zero-mean reverting processes responsible for the spikes or large deviations which revert at a fast rate $\beta_j > 0$.

$X_i(t)$ are zero-mean reverting processes that account for the normal variations in the spot price evolution with lower degree of mean-reversion $\alpha_i > 0$.

The Model

We suppose that the Lévy processes are exponentially integrable in the sense that there exists a constant $\kappa > 0$ such that

$$\int_{|z|\geq 1} e^{\widetilde{\kappa} z}\, \ell_j(dz) < \infty\,, \tag{4}$$

for all $\widetilde{\kappa} \leq \kappa$ and $j = 1, \ldots, n$. This implies that the spot price process $S(t)$ has exponential moments up to order κ, and that the log-moment generating functions defined by

$$\phi_j(x) = \ln \mathbb{E}\left[e^{xL_j(1)}\right]\,, \qquad j = 1, \ldots, n\,, \tag{5}$$

exist for $|x| \leq \kappa$ where ℓ_j is the Lévy measure of the process $L_j(t)$. In the sequel we shall assume that κ is sufficiently large to make the necessary exponential moments of $L_j(t)$ finite.

Indifference Prices

Assume that the producer will deliver the spot over the time interval $[T_1, T_2]$.

He has the choice to deliver the production in the spot market, where he faces uncertainty in the prices over the delivery period, or to sell a forward contract with delivery over the same period.

The producer takes this decision at time $t \leq T_1$.

Indifference Prices

We determine the forward price that makes the producer indifferent between the two alternatives: denote by $F_{\text{pr}}(t, T_1, T_2)$ the forward price derived from the equation

$$
\begin{aligned}
& 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)\, du \right) \mid \mathcal{F}_t \right] \\
= \quad & 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p (T_2 - T_1) F_{\text{pr}}(t, T_1, T_2) \right) \mid \mathcal{F}_t \right]
\end{aligned}
$$

Indifference Prices

Equivalently,

$$F_{\mathrm{pr}}(t, T_1, T_2) = -\frac{1}{\gamma_p}\frac{1}{T_2 - T_1}\ln \mathbb{E}^P\left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)\,du\right) \mid \mathcal{F}_t\right], \quad (6)$$

where for simplicity we have assumed that the risk-free interest rate is zero.

$\int_{T_1}^{T_2} S(u)\,du$ is what the producer collects from selling the commodity on the spot market over the delivery period $[T_1, T_2]$, while he receives $(T_2 - T_1)F_{\mathrm{pr}}(t, T_1, T_2)$ from selling it on the forward market.

Notation

For $i = 1, \ldots, m$ and $j = 1, \ldots, n$,

$$\bar{\alpha}_i(s, T_1, T_2) = \begin{cases} \frac{1}{\alpha_i}\left(e^{-\alpha_i(T_1 - s)} - e^{-\alpha_i(T_2 - s)}\right) & , \quad s \leq T_1\,, \\ \frac{1}{\alpha_i}\left(1 - e^{-\alpha_i(T_2 - s)}\right) & , \quad s \geq T_1\,. \end{cases} \tag{7}$$

and

$$\bar{\beta}_j(s, T_1, T_2) = \begin{cases} \frac{1}{\beta_j}\left(e^{-\beta_j(T_1 - s)} - e^{-\beta_j(T_2 - s)}\right) & , \quad s \leq T_1\,, \\ \frac{1}{\beta_j}\left(1 - e^{-\beta_j(T_2 - s)}\right) & , \quad s \geq T_1\,. \end{cases} \tag{8}$$

Indifference Prices

The price for which the producer is indifferent between the forward and spot market is given by

$$
\begin{aligned}
F_{\mathrm{pr}}(t, T_1, T_2) \quad = \quad & \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du \\
& + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& - \frac{\gamma_p}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\
& - \frac{1}{\gamma_p} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j \left(-\gamma_p \bar{\beta}_j(s, T_1, T_2) \right) \, ds \,,
\end{aligned}
$$

where $\bar{\alpha}_i$ and $\bar{\beta}_j$ are given by (7) and (8) respectively.

Proof – Indifference Price

We calculate the conditional expectation in (6). First observe that

$$\int_{T_1}^{T_2} S(u)\,du = \int_{T_1}^{T_2} \Lambda(u)\,du + \int_{T_1}^{T_2} X(u)\,du + \int_{T_1}^{T_2} Y(u)\,du\,.$$

Proof – Indifference Price

Inserting the explicit dynamics of $X(u)$ and appealing to the stochastic Fubini Theorem, we find

$$\begin{aligned}\int_{T_1}^{T_2} X(u)\,du &= \int_{T_1}^{T_2} \left\{ X(t)\mathrm{e}^{-\alpha(u-t)} + \int_t^u \sigma(s)\mathrm{e}^{-\alpha(u-s)}\,dB_s \right\} du \\ &= X(t)\bar{\alpha}(t, T_1, T_2) + \int_{T_1}^{T_2}\int_t^u \sigma(s)\mathrm{e}^{-\alpha(u-s)}\,dB_s\,du \\ &= X(t)\bar{\alpha}(t, T_1, T_2) + \int_t^{T_2} \sigma(s)\bar{\alpha}(s, T_1, T_2)\,dB_s\,.\end{aligned}$$

Proof – Indifference Price

A similar calculation for $\int_{T_1}^{T_2} Y(u)\, du$ yields,

$$\int_{T_1}^{T_2} Y(u) du = Y(t)\bar{\beta}(t, T_1, T_2) + \int_t^{T_2} \bar{\beta}(s, T_1, T_2)\, dL(s)\,.$$

Proof – Indifference Price

$X(t), Y(t) \in \mathcal{F}_t$, BM and L have independent increments so,

$$\begin{aligned}
&\mathbb{E}\left[\exp\left(-\gamma_{\text{pr}}\int_{T_1}^{T_2} S(u)\,du\right) \mid \mathcal{F}_t\right] \\
&= \exp\left(-\gamma_{\text{pr}}\left(\int_{T_1}^{T_2} \Lambda(u)\,du + X(t)\bar{\alpha}(t, T_1, T_2) + Y(t)\bar{\beta}(t, T_1, T_2)\right.\right. \\
&\quad \times \mathbb{E}\left[\exp\left(-\gamma_{\text{pr}}\int_{t}^{T_2} \sigma(s)\bar{\alpha}(s, T_1, T_2)\,dB_s\right)\right] \\
&\quad \times \mathbb{E}\left[\exp\left(-\gamma_{\text{pr}}\int_{t}^{T_2} \bar{\beta}(s, T_1, T_2)\,dL(s)\right)\right],
\end{aligned}$$

Proof – Indifference Price

$$\mathbb{E}\left[\exp\left(-\gamma_{\text{pr}}\int_{T_1}^{T_2} S(u)\,du\right)\,|\,\mathcal{F}_t\right]$$
$$= \exp\left(-\gamma_{\text{pr}}\left(\int_{T_1}^{T_2}\Lambda(u)\,du + X(t)\bar{\alpha}(t,T_1,T_2) + Y(t)\bar{\beta}(t,T_1,T_2)\right)\right)$$
$$\times \exp\left(\frac{1}{2}\gamma_{\text{pr}}^2\int_t^{T_2}\sigma^2(s)\bar{\alpha}^2(s,T_1,T_2)\,ds\right)$$
$$\times \exp\left(\int_t^{T_2}\phi(-\gamma_{\text{pr}}\bar{\beta}(s,T_1,T_2))\,ds\right).$$

Thus, the Proposition is proved after taking logarithms and dividing by the risk aversion and length of the delivery period.

Indifference Price – Jumps

Suppose $L_j(t)$ is a process of only positive jumps.

Then, the log-moment generating function $\phi_j(x)$ of $L_j(t)$ is an increasing function with $\phi_j(0) = 0$.

Thus, when $x < 0$, $\phi_j(x) < 0$, and since $\bar{\beta}_j$ is positive, we have that the argument of $\phi_j(\cdot)$ in the indifference price of the producer is negative, and thus the jump process $L_j(t)$ causes an increase in the indifference forward price.

Intuitively, positive price spikes work to the advantage of the producer, and he will be reluctant to enter forward contracts that miss such opportunities.

Indifference Price – Jumps

On the other hand, if $L_j(t)$ only exhibit negative jumps, we see that the indifference price is pushed downwards.

Intuitively, the producer is willing to accept lower forward prices since there is a risk of price drops in the spot market.

Indifference Price – Retailer

The retailer will derive the indifference price from the incurred expenses in the spot or forward market, which entails

$$
\begin{aligned}
& 1-\mathbb{E}^P\left[\exp\left(-\gamma_c\left(-\int_{T_1}^{T_2} S(u)\,du\right)\right) \mid \mathcal{F}_t\right] \\
= \; & 1-\mathbb{E}^P\left[\exp\left(-\gamma_c(-(T_2-T_1)F_c(t,T_1,T_2))\right) \mid \mathcal{F}_t\right],
\end{aligned}
$$

or,

$$
F_c(t,T_1,T_2)=\frac{1}{\gamma_c}\frac{1}{T_2-T_1}\ln\mathbb{E}^P\left[\exp\left(\gamma_c\int_{T_1}^{T_2} S(u)\,du\right) \mid \mathcal{F}_t\right]. \tag{9}
$$

Indifference Price – Retailer

The price that makes the retailer indifferent between the forward and the spot market is given by

$$
\begin{aligned}
F_c(t, T_1, T_2) = & \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) \\
& + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& + \frac{\gamma_c}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\
& + \frac{1}{\gamma_c} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j \left(\gamma_c \bar{\beta}_j(s, T_1, T_2) \right) ds\,.
\end{aligned}
$$

Indifference Price – Bounds

Note that the producer prefers to sell his production in the forward market as long as the market forward price $F(t, T_1, T_2)$ is higher than $F_{\mathrm{pr}}(t, T_1, T_2)$. On the other hand, the retailer prefers the spot market if the market forward price is more expensive than his indifference price $F_{\mathrm{c}}(t, T_1, T_2)$. Thus, we have the bounds

$$F_{\mathrm{pr}}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_{\mathrm{c}}(t, T_1, T_2)\,. \tag{10}$$

Market Power

- We introduce the deterministic function $p(t, T_1, T_2) \in [0, 1]$ describing the *market power of the representative producer*.
- For $p(t, T_1, T_2) = 1$ the producer has full market power and can charge the maximum price possible in the forward market (short-term positions), namely $F_c(t, T_1, T_2)$.
- If the retailer has full power, ie $p(t, T_1, T_2) = 0$ (long-term positions), she will drive the forward price as far down as possible which corresponds to $F_{pr}(t, T_1, T_2)$.

Market Power

For any market power $0 < p(t, T_1, T_2) < 1$,
the forward price $F^p(t, T_1, T_2)$ is defined to be

$$\begin{aligned} F^p(t, T_1, T_2) &= p(t, T_1, T_2) F_c(t, T_1, T_2) \\ &\quad + (1 - p(t, T_1, T_2)) F_{pr}(t, T_1, T_2). \end{aligned} \tag{11}$$

Market Power

For $0 \leq t \leq T_1 < T_2$ the forward prices are

$$
\begin{aligned}
&F^p(t, T_1, T_2) \\
&= \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
&\quad + \frac{p(t, T_1, T_2)(\gamma_{\mathrm{pr}} + \gamma_{\mathrm{c}}) - \gamma_{\mathrm{pr}}}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\
&\quad + \frac{p(t, T_1, T_2)}{\gamma_{\mathrm{c}}(T_2 - T_1)} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(\gamma_{\mathrm{c}} \bar{\beta}_j(s, T_1, T_2))\, ds \\
&\quad - \frac{1 - p(t, T_1, T_2)}{\gamma_{\mathrm{pr}}(T_2 - T_1)} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(-\gamma_{\mathrm{pr}} \bar{\beta}_j(s, T_1, T_2))\, ds\,,
\end{aligned}
$$

Risk-Neutral Probabilities

Suppose that we want to price a forward contract with delivery over the period $[T_1, T_2]$. The forward price is defined as

$$F^Q(t, T_1, T_2) = \mathbb{E}^Q\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)\,du \,\middle|\, \mathcal{F}_t\right],$$

where we use F^Q to indicate the dependency on the chosen risk-neutral probability Q.

Risk-Neutral Probabilities

We parameterize the market price of risk by introducing a probability measure $Q^{\theta} := Q_B \times Q_L$, where Q_B is a Girsanov transform of the Brownian motions $B_i(t)$, Q_L is an Esscher transform of the jump processes $L_j(t)$, and θ is an $\mathbb{R}^{n+m}$-valued function describing the market price of risk.

Risk-Neutral Probabilities -Brownian Motions

For $t \leq T$, with $T \geq T_2$ being a finite time horizon encapsulating all the delivery periods in the market, let the probability Q_B have the density process

$$Z_B(t) = \exp\left(- \int_0^t \sum_{i=1}^m \frac{\theta_{B,i}(t)}{\sigma_i(s)} \, dB_i(s) - \frac{1}{2} \int_0^t \sum_{i=1}^m \frac{\theta_{B,i}^2(s)}{\sigma_i^2(s)} \, ds \right) ,$$

where we have supposed that the functions $\theta_{B,i}/\sigma_i$, $i = 1, \ldots, m$, are square integrable over $[0, T]$.

Risk-Neutral Probabilities -Brownian Motions

This measure change in the Wiener coordinates is given by the Girsanov transform,

$$dW_i(t) = -\frac{\theta_{B,i}(t)}{\sigma_i(t)}\,dt + dB_i(t)\,,$$

where $W_i(t)$ become Brownian motions on $[0, T]$, $i = 1, \ldots, m$. The functions $\theta_{B,i}$ represent the compensation market players obtain for bearing the risk introduced by the non-extreme variations in the market, i.e. the diffusion component. We let it be time dependent to allow for variations across different seasons throughout the year.

Risk-Neutral Probabilities -Brownian Motions

This Girsanov change gives the dynamics (for $1 \leq i \leq m$)

$$dX_i(t) = (\theta_{B,i}(t) - \alpha_i X_i(t))\ dt + \sigma_i(t)\, dW_i(t)\,,$$

and thus we have added a time-dependent level of mean-reversion to the processes $X_i(t)$.

Risk-Neutral Probabilities -Lévy Processe

Further, define for bounded functions $\theta_{L,j}$, $j = 1, \dots, n$,

$$Z_L(t) = \exp\left(\int_0^t \sum_{j=1}^n \theta_{L,j}(s)\, dL_j(s) - \int_0^t \sum_{j=1}^n \phi_j(\theta_{L,j}(s))\, ds \right) ,$$

for $t \leq T_2$, and let the density process for the Radon-Nikodym derivative of the measure change in the jump component be

$$\left.\frac{dQ_L}{dP}\right|_{\mathcal{F}_t} = Z_L(t) \, .$$

This is the so-called Esscher transform, and the time dependent functions $\theta_{L,j}(t)$ are the market prices of jump risk.

Risk-Neutral Probabilities

We let $\theta := (\theta_B, \theta_L)$, where $\theta_B := (\theta_{B,i})_{i=1}^m$ and $\theta_L := (\theta_{L,j})_{j=1}^n$.

The density process of the probability Q^θ becomes
$Z(t) := Z_B(t) Z_L(t)$.

We denote by $\mathbb{E}^{Q^\theta}$ the expectation with respect to the probability measure Q^θ.

Forward Price

The forward price $F^{\theta}(t, T_1, T_2)$ is given by

$$
\begin{aligned}
& F^{\theta}(t, T_1, T_2) \\
= \quad & \tfrac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \tfrac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \tfrac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& + \int_t^{T_2} \sum_{i=1}^{m} \theta_{B,i}(s) \tfrac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\, ds \\
& + \int_t^{T_2} \sum_{j=1}^{n} \phi_j'(\theta_{L,j}(s)) \tfrac{\bar{\beta}_j(s, T_1, T_2)}{T_2 - T_1}\, ds\,.
\end{aligned}
$$

for $0 \le t \le T_1 < T_2$.

Forward Price – Proof

The explicit representation of $X(t)$ under Q^{θ} is

$$X(u) = X(t)\mathrm{e}^{\alpha(u-t)} + \int_t^u \theta_B(s)\mathrm{e}^{-\alpha(u-s)}\,ds + \int_t^u \sigma(u)\mathrm{e}^{-\alpha(u-s)}\,dW(s)\,,$$

for $u \geq t$.

Forward Price – Proof

So,

$$\mathbb{E}^{Q^\theta}\left[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S(u)\,du \,\middle|\, \mathcal{F}_t\right]$$

$$= \frac{1}{T_2-T_1}\int_{T_1}^{T_2}\Lambda(u)\,du + X(t)\frac{\bar{\alpha}(t,T_1,T_2)}{T_2-T_1} + Y(t)\frac{\bar{\beta}(t,T_1,T_2)}{T_2-T_1}$$

$$+\frac{1}{T_2-T_1}\int_{T_1}^{T_2}\int_t^u \theta_B(s)\mathrm{e}^{-\alpha(u-s)}\,ds\,du$$

$$+\frac{1}{T_2-T_1}\int_{T_1}^{T_2}\mathbb{E}^{Q_L}\left[\int_t^u \mathrm{e}^{-\beta(u-s)}\,dL_s \,\middle|\, \mathcal{F}_t\right]\,du\,.$$

Forward Price – Proof

Using Bayes and independent increments

$$\begin{aligned}
&\mathbb{E}^{Q_L}\left[\int_t^u e^{-\beta(u-s)}\,dL(s)\,|\,\mathcal{F}_t\right]\\
&=\mathbb{E}^{P}\left[\int_t^u e^{-\beta(u-s)}\,dL(s)\frac{Z_L(u)}{Z_L(t)}\,|\,\mathcal{F}_t\right]\\
&=\mathbb{E}^{P}\left[\int_t^u e^{-\beta(u-s)}\,dL(s)e^{\int_t^u \theta_L(s)\,dL(s)-\int_t^u \phi(\theta_L(s))\,ds}\right]\\
&=\frac{d}{dx}\mathbb{E}^{P}\left[e^{\int_t^u \left(xe^{-\beta(u-s)}+\theta_L(s)\right)\,dL(s)}\right]|_{x=0}\times e^{-\int_t^u \phi(\theta_L(s))\,ds}\\
&=\frac{d}{dx}e^{\int_t^u \phi\left(xe^{-\beta(u-s)}+\theta_L(s)\right)\,ds}|_{x=0}\times e^{-\int_t^u \phi(\theta_L(s))\,ds}\\
&=\int_t^u \phi'(\theta_L)e^{-\beta(u-s)}\,ds\,.
\end{aligned}$$

Risk Premium without Jump Risk

Suppose that the market price of jump risk is zero, i.e. $\theta_{L,j}(s) = 0$ for $j = 1, \ldots, n$. Then

$$F^{\theta}(t, T_1, T_2) = \mathbb{E}^P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)\,du \,\Big|\, \mathcal{F}_t\right] + \int_t^{T_2}\sum_{i=1}^{m}\theta_{B,i}(s)\frac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\,ds\,.$$

Risk Premium without Jump Risk

Thus, we see that when market players are not compensated for bearing jump risk, the market risk premium is positive as long as

$$\pi(t, T_1, T_2) = \int_t^{T_2} \sum_{i=1}^{m} \theta_{B,i}(s) \frac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\, ds$$

is positive.

Risk Premium without Jump Risk

- If all $\theta_{B,i}(t)$'s are positive, then we have a positive market price of risk since $\bar{\alpha}_i$ are positive functions for all $s \leq T_2$.
- In general, one can obtain a change in the sign of the market risk premium over time t by appropriate specification of the functions $\theta_{B,i}(t)$.

Example: Model Specification

We consider a forward market consisting of 52 contracts with weekly delivery. The market power is supposed to be constant $p(t, T_1, T_2) = p \in [0, 1]$. Assume that the spot model has $m = 52$ diffusion components $X_i(t)$, and one ($n = 1$) jump component $Y(t)$. Suppose that the seasonal function is

$$\Lambda(t) = 150 + 20 \cos(2\pi t/365)\,,$$

and the mean-reversion parameters for the diffusion components are $\alpha_i = 0.067/i$, with volatility $\sigma_i = 0.3/\sqrt{i}$, for $i = 1, \dots, 52$.

Model Specification

- We mimic here a sequence of mean-reverting processes with decreasing speeds of mean reversion and with decreasing volatility.
- The speed of mean reversion equal to 0.067 means that a shock will be halved over 10 days.
- The jump process is driven by $L(t) = \eta N(t)$, where $N(t)$ is a Poisson process with intensity λ and the jump size is constant, equal to η.

Model Specification

- The mean-reversion for the jump component is $\beta = 0.5$, meaning that a jump will, on average, revert back in two days.
- We have a combination of slow mean reverting normal variations and fast mean reverting spikes in the spot market.
- The frequency of spikes is set to $\lambda = 2/365$, i.e. two spikes, on average, per year.

Model Specification

- Time $t = 0$ corresponds to January 1, and we assume that the initial spot price is $S(0) = 172$.
- We let $X_1(0) = 2$, and $X_i(0) = Y(0) = 0$ for $i = 2, \ldots, 52$ to achieve this.
- The risk aversion coefficients of the producer and retailer are set equal to $\gamma_c = \gamma_{pr} = 0.5$.
- We derive forward curves for weakly settled forward contracts over a year.

Indifference price with forward curves for positive jumps

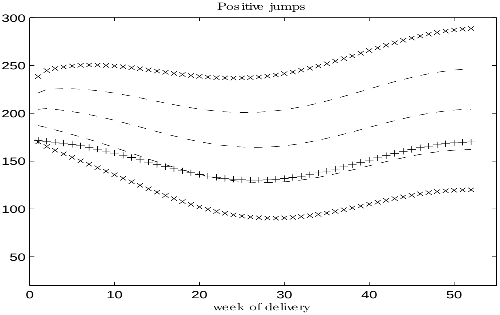


Figure 1: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted '+'. The jumps are positive of size 10.

Market risk premium – positive jumps

- Market clearing forward prices are increasing with increasing market power, since the producer will command higher prices with more power.
- For a low market power of 0.25, we observe that the forecasted price curve is below the forward curve in the shorter end, while in the medium to long end we see the opposite.
- This corresponds to a positive market risk premium in the shorter end, whereas it becomes negative in the medium and longer end.
- The retailer wishes to avoid upward jumps in the price and is, even for a weak producer, willing to accept a positive market risk premium in the short end. In the long end, the effect of jumps vanish as a consequence of mean reversion, so the retailer will have more power.

Market risk premium – positive jumps

To illustrate this particular example we have plotted the difference of the forward curve with market power 0.25 and the forecasted curve. For the contracts with delivery up to approximately week 20, the market premium is positive. The premium decreases with time to delivery, and becomes negative in the medium and long end.

Market risk premium – positive jumps

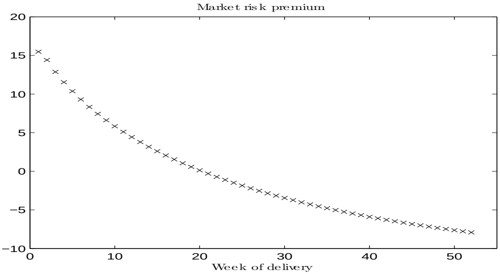


Figure 2: The market risk premium given by the difference of the forward curve with market power 0.25 and the forecasted curve.

Indifference price curves forward curves for negative jumps

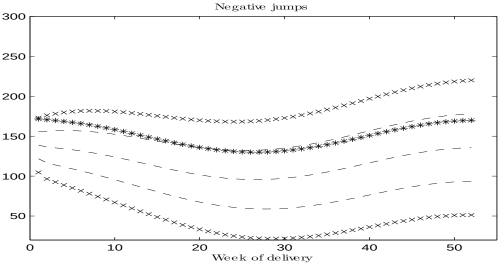


Figure 3: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted with '*'. The jumps are negative of size 10.

Market risk premium – negative jumps

- We observe that all curves are shifted downwards, indicating that the producer is willing to accept lower forward prices to hedge the possibility of sudden drops in prices.
- In the short-term we observe, for all cases of market power, that the forecasted spot price is above forward prices, i.e. negative market risk premium.
- In the long-term, only when producer's market power is high, that is 0.75, we have the situation where the forecasted curve is below the forward curve signaling that the retailer bears a positive risk premium.

Market risk premium – negative jumps

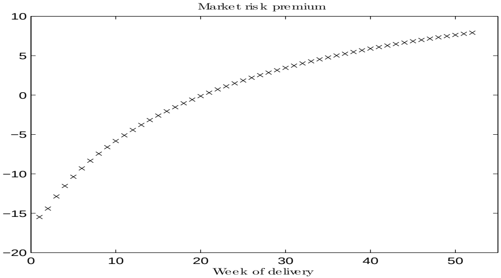


Figure 4: The market risk premium given by the difference of the forward curve with market power 0.75 and the forecasted curve.

Estimation problems

- We need to estimate the physical parameters of our two-factor model.
- From forward market data, denoted by $F(t, T_1, T_2)$, we estimate the risk-aversion coefficients for both producers and retailers and estimate the producer's market power.

Data used

- Spot prices: Phelix base load traded at the EEX.
- Forward contract prices with delivery periods: monthly, quarterly and yearly.
- Period covered: January 2 2002 to January 1 2006 with 1461 spot price observations.
- Forward data: 108 contracts with monthly delivery, 35 contracts with quarterly delivery and 12 contracts with yearly delivery.

Spot model specification

We apply the model to

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

where, $\Lambda(t)$ is the seasonal component,

$$dX(t) = -\alpha X(t)dt + \sigma dB(t) \tag{12}$$

where $\alpha \geq 0$, $\sigma \geq 0$ and $B(t)$ is a standard Brownian motion,

Spot model specification

$$dY(t) = -\beta Y(t)dt + dL(t) \tag{13}$$

with $\beta \geq 0$ and

$$L(t) = \sum_{i}^{N(t)} J_i \tag{14}$$

is a compound Poisson process.
$N(t)$ is a homogeneous Poisson process with intensity λ and J_i's are i.i.d. with exponential density function

$$f(j) = p\lambda_1 e^{-\lambda_1 j} \mathbf{1}_{j>0} + (1-p)\lambda_2 e^{-\lambda_2 |j|} \mathbf{1}_{j<0},$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$ are responsible for the decay of the tails for the distribution.

Spot model specification

For the seasonal component we assume

$$\begin{aligned}\Lambda(t) &= a_0 + a_1 \mathbf{1}_{\{t=Su\}} + a_2 \mathbf{1}_{\{t=Mo,Fri\}} + a_3 \mathbf{1}_{\{t=Tu,We,Th\}} + a_4 \mathbf{1}_{\{t=\\ &\quad + a_5 \cos\left[\frac{6\pi}{365}(t + a_6)\right] + a_7 t,\end{aligned}$$

where the indicator function is acting on the different days of the week.

Risk aversion coefficients

Recall that $F_c(t, T_1, T_2)$ (upper bound) and $F_{pr}(t, T_1, T_2)$ (lower bound) depend on the choice of γ_c and γ_{pr}, we estimate γ_{pr} and γ_c by minimizing the distance between $F_c(t, T_1, T_2)$, $F_{pr}(t, T_1, T_2)$ and the market prices of forwards $F(t, T_1, T_2)$, respectively, in the following way.

Risk aversion coefficients

- For all trading days $t \in [1, 1461]$, we determine all values of γ_{pr} and γ_c such that

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2). \qquad (15)$$

- We define the intervals I_{pr}^t and I_c^t containing values for γ_{pr} and γ_c by guaranteeing that (15) holds.
- For the intersection of all these interval no forward prices $F(t, T_1, T_2)$ will lay outside the bounds $F_{pr}(t, T_1, T_2)$ and $F_c(t, T_1, T_2)$.
- We find that $\gamma_{pr} \in [0.421, \infty)$ and $\gamma_c \in [0.701, \infty)$.
- Thus we choose $\gamma_{pr} = 0.421$ and $\gamma_c = 0.701$.

Market power and market risk

Recall

$$p(t, T_1, T_2) = \frac{F(t, T_1, T_2) - F_{pr}(t, T_1, T_2)}{F_c(t, T_1, T_2) - F_{pr}(t, T_1, T_2)}$$

and

$$\pi(t, T_1, T_2) = F(t, T_1, T_2) - \mathbb{E}^P \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u) du | \mathcal{F}_t \right].$$

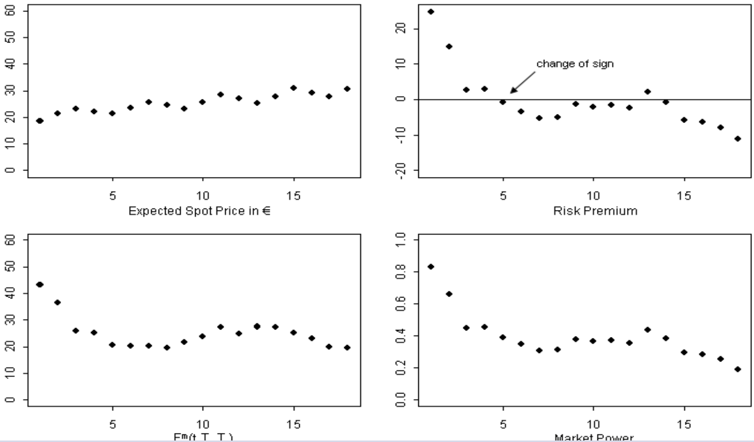
Market power and market risk

We consider three periods

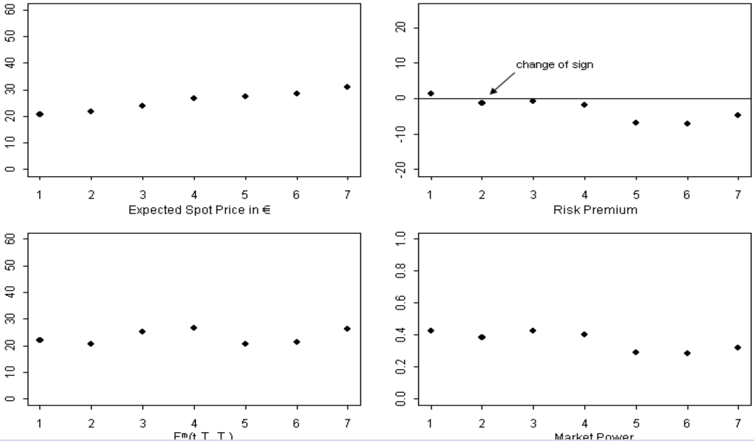
t	Type	# Contracts	Delivery Periods
01/Jan/2002	monthly	18	Jan 2002 - May 2003
01/Jan/2002	quarterly	7	2nd qtr 2002 - 4th qtr 2003
01/Jan/2002	yearly	3	2003 - 2005
03/Mar/2003	monthly	7	Feb 2003 - Aug 2003
03/Mar/2003	quarterly	7	2nd qtr 2003 - 4th qtr 2004
03/Mar/2003	yearly	3	2004 - 2006
04/Oct/2005	monthly	7	Oct 2005 - Apr 2006
04/Oct/2005	quarterly	7	1st qtr 2006 - 3rd qtr 2007
04/Oct/2005	yearly	6	2006 - 2011

Table: Forward contracts

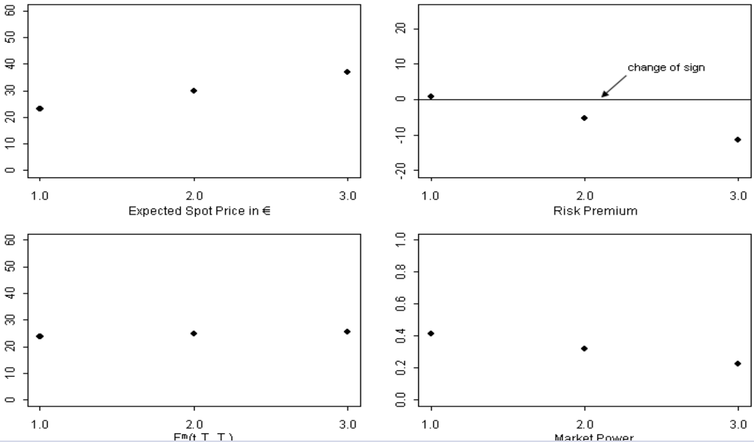
Producer's market power and market risk premium, 18 monthly contracts with t = January 2 2002



Producer's market power and market risk premium, 7 quarterly contracts with $t =$ second quarter 2002



Producer's market power and market risk premium, 3 yearly contracts with $t = 2002$

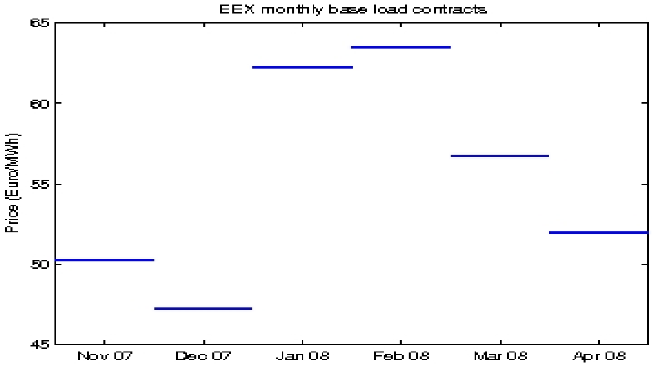


Agenda

Market Risk Premium – Information Approach

- Since electricity is non-storable future predictions about the market will not affect the current spot price, but will affect forward prices.
- Stylized example: planned outage of a power plant in one month
- Market example: in 2007 the market knew that in 2008 CO_2 emission costs will be introduced; this had a clearly observable effect on the forward prices!

Information Approach – Market Example



Information Approach – Definition

- Define the forward price as

$$F_{\mathcal{G}}(t,T) = \mathbb{E}[S(T)|\mathcal{G}_t]$$

- $\mathcal{G}_t$ includes spot information up to current time ($\mathcal{F}_t$) and forward looking information
- The information premium is

$$I_{\mathcal{G}}(t,T) = F_{\mathcal{G}}(t,T) - \mathbb{E}[S(T)|\mathcal{F}_t].$$

- Theoretical analysis uses the theory of enlargements of filtrations