Book review

Handbook of Tableau Methods, edited by Marcello D'Agostino, Dov M. Gabbay, Reiner Hähnle, Joachim Posegga

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Introduction

As an undergraduate student I bought myself a copy of the *Handbook of Mathematical Logic* (Barwise, 1978). I found it to be an invaluable resource and an essential entry point to the literature. Since then, I have used handbooks and similar publications in the area of logic on many occassions; these include the *Handbook of Philosophical Logic* (Gabbay and Guenthner, 1989), the *Handbook of Theoretical Computer Science* (van Leeuwen, 1990), the *Handbook of Logic in Artificial Intelligence and Logic Programming* (Gabbay et al., 1998), the *Handbook of Logic in Computer Science* (Abramsky et al., 2000), the *Handbook of Logic and Language* (van Benthem and ter Meulen, 1997), the *Handbook of Formal Languages* (Rozenberg and Salomaa, 1997), and the *MIT Encyclopedia of the Cognitive Sciences* (Wilson and Keil, 1999),. Such publications are best viewed as 'portals' for the disciplines they're covering, providing easy access to areas in which one is not an expert and giving comprehensive overviews of areas in which one is.

So now there is the *Handbook of Tableau Methods.* The editors motivate the need for this handbook by pointing out that, recently, interest in tableaus has become more widespread and that a community has crystallized around the topic. An annual tableaus conference is being held, and proceedings are published.

But what are tableau methods? A tableau method is a formal proof procedure with certain characteristics. First, it is a refutation procedure: to show a formula ϕ is valid we begin with some syntactical expression intended to assert it is not. How this is done is a detail, and varies from system to system. Next, the expression asserting the invalidity of ϕ is broken down syntactically, generally splitting things into several cases. This part of a tableau procedure — the tableau expansion stage — can be thought of as a generalization of disjunctive normal form expansion. Generally, it involves moves from formulas to subformulas. Finally, there are rules for closing cases: impossibility conditions based on syntax. If each case closes, the tableau itself is said to be closed. A closed tableau beginning with an expression asserting that ϕ is not valid, is tableau proof of ϕ.

There is a second, more semantical, way of thinking about the tableau method, one that has played a lesser role thus far: it is a search procedure for models meeting certain conditions. Each branch of a tableau can be considered to be a partial description of a model. Several fundamental theorems of model theory have proofs that can be extracted from results about the tableau method. (Smullyan, 1968) developed this approach, and it was carried further by (Bell and Machover, 1977). In automated reasoning, tableaus are sometimes used to generate counter-examples. The connection between the two roles for tableaus — as a proof procedure and as a model search procedure — is simple. If we use tableaus to search for a model in which ϕ is false, and we produce a closed tableau, no such model exists, so ϕ must be valid.

'The present volume is a *Handbook of Tableaux* (sic!) presenting to the community a wide coverage of tableaux systems for a variety of logics' (page vii).

Contents

What's in the *Handbook of Tableau Methods*? We get a two page preface, an introduction by Melvin Fitting, eight specialized chapters on tableaus for specific logics, a chapter on implementing tableaus, and a bibliography on analytic tableaus theorem proving. To top it off, there is an extensive index to the handbook. Some of the chapters overlap considerably in their contents. According to the editors, '[t]his was a deliberate choice, motivated primarily by the need for making each chapter selfcontained.'

Let's take a closer look at the individual chapters now.

M. Fitting: Introduction. The first chapter contains a general introduction to the subject which can help the reader in finding a route through the following chapters, but can also be read as a 'crash course' on tableaus, concentrating on the key ideas and the historical background. To make matters concrete, Fitting discusses syntactical means for asserting invalidity, and syntactic means allowing a case analysis. In addition, machinery is needed for closing cases. All this is (obviously) logic dependent, and Fitting gives examples of several kinds throughout his chapter.

Tableau history essentially begins with Gentzen. For classical logic, ignoring issues of machine implementation, it culminates with Smullyan's work. Fitting discusses this portion of the development of the subject in Section 2 of his chapter. The third section is devoted to the extension of the tableau method to non-classical logics, and the fourth to the (history of the) automation of tableaus. The chapter provides an extremely interesting account of some aspects of the history of tableaus, with lots of connections and facts that I was not aware of.

Unfortunately, the chapter already shows its age. All references — except one — are to publications from 1993 or before, and the only exception is a reference to the 1996 edition of the author's *First-Order Logic and Automated Theorem Proving.*

M. D'Agostino: Tableau Methods for Classical Propositional Logic. The author explores and compares the main types of tableau methods which appear in the literature, paying special attention to variants and 'improvements' of the original method. After having introduced valuations and some basic notions in computational complexity, the author goes on to discuss Smullyan's tableaus and its links to natural deduction and resolution. The next section is devoted to extensions of Smullyan's tableaus that aim to tackle some of the shortcomings that the latter have for proof search. In particular, it is shown how proofs in Smullyan's calculus may generate many (and large) redundant subproofs. The proposed additions to Smullyan's tableaus are meant to prevent this; note, however, that the additions are *redundant* in that they can be left out without destroying completeness of the calculus.

The longest section in the chapter is mainly devoted to the introduction and discussion of a tableau calculus that avoids the inefficiency of Smullyan's calculus without having redundant proof rules: the system **KE** first proposed by Mondadori in 1988. All its expansion rules except one are linear rules, that is, rules which do not force branching. The only branching rule is the principle of bivalence, which allows one to branch at any point in a proof, with any formula A, to either A or $\neg A$. The pros and cons of **KE** are discussed extensively, and it is shown that Smullyan's tableaus cannot polynomially simulate **KE**.

R. Letz: First-Order Tableau Methods. This chapter contains an investigation of the impact of tableaus on the challenging problems of classical quantification theory. The chapter consists of 6 sections, the first two of which cover the basics of first-order logic and normal form transformations. A formulation of tableaus for first-order logic is presented in the third section, while the fourth concentrates on key weaknesses of traditional tableau systems from the point of view of proof search. The main problem is the choice of instantiations in expansion rules for universally quantified formulas and negated existentially quantified ones. In traditional tableaus this choice is often done too early; free-variable tableaus attempt to remedy this by allowing free variables in a tableau which are treated as placeholders for terms, as so-called 'rigid' variables. The instantiation of rigid variables is guided by unification. Unfortunately, systematic procedures for building free-variable tableaus cannot be deviced as easily as for traditional 'closed formula' tableaus; therefore, various enumeration procedures are used instead.

The last two sections of the chapter are devoted to tableaus for clause logic (which admit a condensed representation of tableaus) and to methods for shortening tableau proofs.

B. Beckert: Equality and Other Theories. The fourth chapter takes a look at various methodologies for equality reasoning. Theory reasoning is indispensable for automated deduction in real world domains. While efficient equality reasoning is especially important, most specifications of real word problems use other theories as well: algebraic theories in mathematical problems and specifications of data types in software verification, to name a few.

In 10 sections (varying in length between half a page and 11 pages) the author presents an overview of how to design the interface between semantic tableaus (the foreground reasoner) and a theory background reasoner. The problem of handling a certain theory is reduced to finding an efficient background reasoner for that theory. In particular, for handling equality a number of specialized methods are discussed in the chapter. The most efficient of these are based on so-called E-unification techniques. Just like the general problem of designing background reasoners is difficult to solve in a uniform way, so, it turns out, is the problem of developing E-unification procedures.

A. Waaler, L. Wallen: Tableaux for Intuitionistic Logics. The treatment of tableaus for non-classical logics is taken up in this chapter, which deals with intuitionistic logic. The chapter starts by recalling Heyting's definition of meaning of the intuitionistic connectives via proof interpretations and Kripke's alternative semantic scheme for intuitionistic logic. Section 2 is the technical heart of the chapter. The authors formulate a system LB (after the Dutch logician Beth) which is a notational variant of Fitting's tableau system for intuitionistic logic. The motivation for the system LB mostly comes from considerations on proof search.

The third and final section of the chapter is devoted to optimizations aimed at pruning the search space. The authors focus on two issues in particular: (i) restrictions on propositional and predicate logic, and (ii) the treatment of first-order quantification using ideas going back to Herbrand, Skolem, and Robinson.

R. Goré: Tableau Methods for Modal and Temporal Logics. An area in which tableau methods have proved particularly useful is modal logic. The increasing use of modal and modal-like logics in areas as diverse as cryptography, economy, and knowledge representation has given rise to an increase in attention to (automated) reasoning methods for modal logic. Indeed, whereas resolution reigns supreme in automated reasoning for first-order logic, tableaus is the preferred method in automated reasoning for modal logic.

In this chapter, the author focuses on the logical and mathematical foundations of modal tableaus, largely ignoring implementational aspects. The core section of the chapter is the fourth one, which consists of 21 subsections. In it, Goré discusses everything from motivations, introductory technicalities, and relations of his own calculi to systems of Fitting and Smullyan to proof theoretic issues (like structural, admissible, and derivable rules) and techniques for proving soundness and completeness results. Goré then goes on to discuss specific tableau systems for epistemic, provability, and temporal logic. The final part of the section covers the connection between modal tableau systems and modal sequent systems.

In the last two sections of the chapter, Goré presents tableau methods for multi-modal logics as well as labeled modal tableau systems where labels attached to formulas are used to keep track of the states in the tableau construction.

M. D'Agostino, D.M. Gabbay, K. Broda: Tableau Methods for Substructural Logics. A further area of particular interest for tableau methods is the area of substructural logics, which include relevance logic and linear logic. The authors focus on two main lines of research: the approach based on 'proof-theoretic' tableaus developed by McRobbie, Belnap and Meyer (which is motivated by work done in the tradition of relevance logic), and the approach based on labeled tableaus, which builds on Gabbay's research program on labeled deductive systems.

About a dozen pages are devoted to the former approach, which is discussed in an informal, discursive manner. The labeled approach receives much more attention: over 65 pages. Most of the exposition revolves around a labeled generalization **LKE** of the system **KE** that was discussed in Chapter 2 of the *Handbook*, and many examples are given to illustrate the main technical definitions and results.

N. Olivetti: Tableaux for Nonmonotonic Logics. Tableau methods are one of the few proof formats that have been successfully used in nonmonotonic reasoning. The author identifies two types of approaches to nonmonotonic reasoning: the *fixpoint approach* and the *semantic preference approach*. The former covers all proposals in which nonmonotonic inferences are sanctioned by non-provability. The latter (which is also referred to as the minimal entailment approach) is based on the idea of restricting the notion of logical consequence to a subset of minimal or preferred models of the axioms.

The second and third section of the chapter are devoted to the fixpoint and semantic preference approach, respectively. After that we get a brief section on tableaus as a general methodology, which serves as a preparation for presenations of tableaus for autoepistemic logic, default logic, and minimal entailment. All of these discussions are restricted to the case of propositional logic; in the penultimate section of the chapter the author discusses tableau methods for nonmonotonic reasoning with first-order logic. The last couple of pages of the chapter are devoted to recent developments in the area.

R. Hähnle: Tableaux for Many-valued Logics. In recent years, many-valued logics have re-gained interest in the research community. Reiner Hähnle offers a wealth of concepts and results. The chapter starts out with a brief discussion of many-valued logic, an overview of the basic notions, and a discussion early work on proof methods for many-valued logic. In Chapter 2 of the *Handbook* it was shown that classical (signed) tableau systems correspond in a one-to-one manner to cut-free sequents; in Section 4 of the present chapter this correspondence is extended to many-valued sequent systems and many-valued (signed) tableaus. Section 5 is devoted to alternative presentations of many-valued logic, for instance in terms of mixed integer programs. Next come an exposition on efficient deduction in many-valued logic and one on connections to other formalisms (such as binary decision diagrams) and on applications.

J. Posegga, P. Schmitt: Implementing Semantic Tableaux. The authors of this chapter present a 'minimalist approach to the implementation of classical tableaux' (page viii). The authors present executable code for the $\mathsf{lean}T^AP$ theorem prover. The idea behind $\mathsf{lean}T^AP$ is to implement logical calculi by minimal means. This has two advantages: first, the resulting programs are small, which makes it easier to understand them, and second, they provide an ideal starting point for further work as they can easily be modified or adapted to specific needs.

The first couple of sections in this chapter discuss some preliminaries concerning both Prolog (the language in which $\mathsf{lean}T^AP$ is implemented) and normal form transformations. Section 3 presents the first and simplest version of $\mathsf{lean}T^AP$: just 5 clauses; both soundness and completeness are proved for the program. Section 4 proposes some heuristics, based on so-called *universal formulas.* Subsequent sections discuss alternative presentations of the method and the use of lemmas.

G. Wrightson: A Bibliography on Analytic Tableaux Theorem Proving. This chapter is advertized as 'an extensive annotated bibliography' (page viii). It consists of 270 references divided into several categories: Early Work, Books and Proceedings, Classical Logic, Non-Classical Logic, and Implementations. Unfortunately, the bibliography is already fairly dated (for instance, it does not mention the proceedings of the annual tableau workshop that have appeared after 1995) and it has many ommissions (for instance, there are no references on what I think has been the source of a lot of important and innovative work on tableaus for modal and modal-like logics over the past decade: description logic).

Despite these criticisms, the bibliography will prove to be a valuable resources for anyone interested in tableaus. It is to be hoped that someone makes it available on-line in a way that will enable anyone in the community to suggest additions.

Evaluation

While my description of the *Handbook of Tableau Methods* should convey the message that this book is very rich in content, there are various topics whose inclusion would have increased the value of the *Handbook.* For instance, there's no systematic treatment of the use tableaus for obtaining complexity results. There's no systematic proof-theoretic treatment of tableaus vs. axioms vs. natural deduction vs. Gentzen-type calculi vs. resolution. There's hardly any discussion on work done in the description logic community, where tableaus have been the method of choice for over a decade. There's no discussion of links with automata-based decision procedures, which is particularly relevant in areas such as model checking and temporal logic. There's no systematic discussion of tableaus vs. other methods in the area of propositional satisfiability checking; see (Gent et al., 2000)...

Another complaint I have concerns the presence of an unacceptably large number of typos and, what appear to be, LaTeX errors, especially in Chapter 5 (by Waaler and Wallen). A book which (I suppose) is meant to become a standard reference, deserves more careful copy-editing.

So there we have it: 680 pages packed with valuable information on tableaus. I would advice anyone interested in tableaus to consider purchasing this book. But this book comes at a price, and a considerable one at that. To be precise, the *Handbook of Tableau Methods* costs $297.00. That's just under 44 cents per page, which makes it one of the most expensive handbooks that I have seen in a long time.[1] I know of several libraries that have decided *not* to purchase the *Handbook of Tableau Methods* because of its outrageous price. Thus, despite my earlier advice to consider purchasing it, I doubt whether anyone will be in a position to do more than just that: *considering* a purchase.

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[1] To give this review a thoroughly Dutch twist, here's a brief overview of the costs of the handbooks and encyclopedia mentioned in the first paragraph of this review:

Title	Publisher	No. of pages	Price (US$)	Price per page (US$)
Hndbk. of Mathematical Logic[a]	Elsevier	1178	113.00	0.096
Enc. of the Cognitive Sciences[b]	MIT Press	1312	149.95	0.114
Hndbk. of Logic and Language[a]	Elsevier	1272	164.00	0.129
Hndbk. of Formal Languages[c]	Springer	2100	285.00	0.136
Hndbk. of Theoretical CS[a]	Elsevier	2284	468.00	0.205
Hndbk. of Logic in AI and LP[d]	Oxford UP	3072	760.00	0.247
Hndbk. of Logic in CS[d]	Oxford UP	3160	1050.00	0.332
Hndbk. of Tableau Methods[e]	Kluwer	680	297.00	0.437
Hndbk. of Philosophical Logic[e]	Kluwer	2556	1436.50	0.562

[a] Information obtained from `www.elsevier.com` on December 6, 2000.
[b] Information obtained from `mitpress.mit.edu` on December 6, 2000.
[c] Information obtained from `www.amazon.com` on December 6, 2000.
[d] Information obtained from `www.oup-usa.org` on December 6, 2000.
[e] Information obtained from `www.wkap.nl` on December 6, 2000.

Except for the *Handbook of Mathematical Logic* all prices are for hard cover editions.

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