Outstanding Contributions to Logic 5

Alexandru Baltag Sonja Smets Editors

Johan van Benthem on Logic and Information Dynamics



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Preface by Johan van Benthem

This book appears in a series highlighting contributions to logic, as seen through the eyes of a congenial community of colleagues. It is a great honor to be a focus for the group of authors assembled here. Though public mirrors seldom reflect selfimages, we learn most about people, not only by their own words, but also by the company they keep.

But enough said about people, let me turn to the topic. This book is about *logical dynamics*, a bundle of interests and a program that may not cover my whole work, but that definitely constitutes the largest chunk of what I have done over the last decades. Let me explain what it means to me. You may find what follows ideological, some people prefer context-free theorems—but I need such broader perspectives even for myself, to remind me of why I do the things I do—or even, why I do research at all.

The main idea of logical dynamics is the pervasive duality between information-related actions and their products. Standard logical systems emphasize notions like formula or proof in the sense of static objects that can be viewed or even manipulated externally. But these objects are produced in activities of communicating statements, engaging in reasoning, and many other intellectual skills. Interestingly, our natural language is often ambiguous in this respect between verbs or other activity-related expressions and static nouns. A dance is an activity I can engage in, but also an object that can be produced by dancing—and the same duality holds for many logical terms, like "statement" or "argument." The idea of logical dynamics is to take this duality seriously, and bring the core logical activities explicitly into formal systems that satisfy the same standards of rigor as the ones that we know and love. This is possible, since activities and events, too, have a formal structure that lends itself to logical analysis. In this way, to borrow a happy phrase, "logic can be more than it is."

Over time, my view of what are logical core activities has evolved from singleagent acts of inference and observation to social scenarios involving more agents, with asking a question, perhaps, as the major instance of a basic logical act. This took some time, since this social turn went against central tenets of my upbringing. Dutch Calvinists like me were raised with the idea that there are two modes of life. The horizontal mode looks at other people and what they think, the vertical one looks only at one's relationship to God. Naturally, the latter, more lonely but also more heroic stance appealed much more to me, and logic seemed very much in that spirit, putting one in direct communion with the intellectual joints of the universe. By contrast, the horizontal stance is all about being influenced by and dependent on others, that is, the realm of human frailty and folly. But over time, I have come to appreciate that social behavior and the intricate network of dependencies that form our life may be the more exciting and challenging phenomenon—or at least, that it has equal importance to solitude in logic and intellectual life, just as the various interactions of particles that constitute our physical world. In fact, perhaps the original source for logic is argumentation between different parties, with formal systems coming only later as a methodological device. And again, logic can deal with all these perspectives at once.

Formal versions of these views in their various phases can be found in a sequence of my books: *Language in Action* (1991), *Exploring Logical Dynamics* (1996), *Logical Dynamics of Information and Interaction* (2011), and *Logic in Games* (2013). Another important source is the dissertations of my students since roughly 2000. What all these publications reflect are influences on my thinking from the worlds that meet in my academic environments at Amsterdam and Stanford: logics of action and processes in computer science, dynamic semantics of natural language, philosophical theories of knowledge and information, and interaction as studied in game theory. I see logic as lying at a crossroads of the university, absorbing many ideas that pass.

Still, being a logician also implies a certain modus operandi, and in my view, a unity of methods persists even when we expand the agenda: logical dynamics uses formal systems. In much of my work, systems of modal logic play an important role, as a convenient light formalism that allows us to see a lot of interesting structures without importing too much machinery. But I see exclusive allegiance to one formalism or school as an intellectual weakness, and I have in fact devoted a lot of time to seeing connections and parallels between different logical systems, as in my work on correspondence theory. Still, the main point is the formal slant in this kind of work per se. Even when I theorize about noisy "horizontal" social reality, the methodology is "vertical," the mathematical truth is absolute, and social strategizing would not help.

So much for my own take on the topic of this book. But a book like this is a risk, since it is a mirror in one's colleagues' eyes, who may see things quite differently. Sometimes you wish you were the person portrayed, sometimes the mirror confirms your worst suspicions. That is why so many people with books devoted to their work are engaged in frantic spin covering the entries with added responses, conclusions, and other types of cotton candy. I will try to minimize this spin mode, though I cannot refrain from making a few points about the book as I experience it—both the editorial process of producing it, and the product that now lies before us.

For a start, though one can have lots of soul-searching thoughts, at a most simple and immediate level, this book just consists of topics that I like! Many chapters represent some aspect of logic, information, and agency that I would like to understand better—and that is exactly what the authors have provided. Moreover, I admit to just liking abstract technical logic, and again many authors have done just that, stepping up their abstraction levels in those typical ways that please logicians. I will not even begin to enumerate all chapter topics here: a later separate piece will present some more detailed thoughts concerning what the authors have to say. But even so, it will be clear that this book contains many trails of happy hiking in the landscape of logical dynamics, very broadly conceived. Some of these trails start out in places where I have walked myself these past decades, such as dynamic epistemic logics, temporal logics, logics of games, or belief revision—while other trails in the book move out into relatively new territory for me, such as learning theory, social dynamics, database theory, proof theory, cognitive science, or probability theory.

But the material collected here defies easy description. What also appeals very much to me is chapters that remind me of my earlier interests in natural language, philosophical logic, and philosophy of science. They made me realize that there may be much more continuity of concerns than I have perhaps thought over the past period, and many more things to be learnt by returning there, than I had imagined. Likewise, there is material on my old and persistent technical interests in modal model theory and foundations of computation that I find extremely suggestive, especially, as I feel that applied agenda extensions for our discipline, as envisaged in logical dynamics, had better be accompanied by rigorous theoretical investigations from the start.

Of course, not every author approaches things the way I myself would do it: I guess this realization on my part is the process called learning. In fact, on a selfcritical note, several chapters have taught me that logical dynamics is not such a clear concept as I would like to think. There are serious philosophical issues about its precise claims and its relationship to classical logics, and there are mathematical issues about a best understanding of how its dynamic systems had best be formulated and understood. Much of this has generated lively correspondence with authors, and I hope that some of this ongoing discussion will itself find its way into the literature.

In order to give this book its present focus, selections had to be made. Some loves from my earlier life do not occur, or not enough justice is done to them, such as the interfaces of logic with natural language, philosophy, and cognition described in my scientific autobiography. This restricted focus is the format of this series, and I think it is inevitable for any readable book. Still, several authors have made connections to these other topics that set me thinking. I now feel that natural language is much more important to logical dynamics than I had realized so far, while there is also a clear potential for revitalizing the interface of logic and philosophy. And even cognitive science is just around the corner: while my systems of logical dynamics remain normative, they can only function in the real world. On the sunny side, even gaps and loose ends that come to light tell me where I might be going from here. For instance, I find myself drawn increasingly to interfaces between logic and probability, and many chapters in this book whet that appetite. I find this a comforting thought. Although a book series like this new initiative might be considered a polite invitation to outstanding logicians to finally shut up and leave the field to a younger generation, I see some rays of future for me shining through its pages.

I find it hard to tell other people what sort of book this is. It is not a Festschrift, it is not just an anthology, it has no systematically enforced message or methodology. It is much more ambiguous than that, like life itself. The way I experience this book, it is a panorama of a world I enjoy. It demonstrates the broad interests and methods that have shaped my own work in logic. But I hope it does not do that too obtrusively. Even if you are not into logical dynamics (or Johan van Benthem), the pieces that follow should still be of interest. Their topics are important, and represent a future for logic. Moreover, the group of their authors itself conveys an important message. They come from many disciplines: mathematics, philosophy, computer science, artificial intelligence [the love child of computer science and philosophy], but also game theory and beyond. This diversity is my world where I feel comfortable, this is how I was educated, and how my academic environment functions. I deeply feel that the broad logic that is at stake here can only flourish in this sort of intellectual company.

Thanks to the authors for contributing what they did, and lending their presence to this book. And thanks to the editors Alexandru and Sonja for making it happen.

Johan van Benthem



ADDENDUM II The Life of Logic, a Scientific Autobiography

How does one end up as a logician? Choice problems have been a constant companion in my life, starting in my gymnasium days. I loved classical languages and history, but also mathematics and the sciences. In those days, one had to choose one type or the other eventually, but I managed to beat the system. I did my official school exam in the 'beta' science track, but with the help of extra lessons after classes, I also took a parallel national exam in the 'alpha' language track. With those two degrees in hand, I still found myself without any preference for a field of study at the university, so I took physics, since people told me it is the hardest discipline, and best for keeping your brain active while waiting for inspiration to strike. And then I ran into logic. A fellow student who had observed that I always managed to talk myself into a corner in discussions suggested I should read a logic book to find out what was wrong with me. As it happened, it was a 19th century text by William Stanley Jevons, which had been translated into a popular Dutch pocket book series. That chance encounter set my course: I was intrigued by the subject, and switched to studying mathematics and philosophy, as the two obvious companion disciplines.

Well, this is the official story. I did have one very specific burning ambition at age 18, to become a literary author. I had collected all my heartfelt short stories and sent them to a well-known Dutch publisher. The answer was that there was a little merit, and a lot of adolescent immaturity, and I was advised to submit again in some 20 years. I got the point. Disappointments guide our lives more firmly than fond hopes.

My interest in logic had some features of a spiritual conversion. I remember the feeling of enlightenment coming from realizing that there are mathematical patterns behind the daily stream of our language and reasoning. That feeling was much reinforced by the organized religion behind this spiritual experience. Reading Nagel and Newman's book *Gödel's Proof* was like entering a world of holy gospel.

My life as a student was at the intersection of philosophy and mathematics. The logic students and teachers formed a truly interdisciplinary team, and I was lucky to see a golden generation in action, with people such as Dick de Jongh, Hans Kamp, Anne Troelstra, Wim Blok, Peter van Emde Boas, and others, including my supervisor Martin Löb. And there was of course that mysterious thing called the international community. I still remember the feeling of anticipation and then fulfillment when

picking up a blue airmail envelope from a logician in the United States or New Zealand. Vanished pleasures! For me, modal logic was the ideal bridge between philosophy and mathematics, combining the best of both: mathematical challenge and conceptual motivation. I well remember the excitement of those early days, with lots of new questions floating around each week. Of course, there was also the intense pressure of having to start doing creative work on a par with these formidable others. One such experience that I remember vividly concerns my proof (by proving theorems logicians are really trying to prove themselves) that the McKinsey Axiom is not first-order definable on modal frames. In that period, I once had to spend a week in an Amsterdam hospital in the aftermath of surgery, and as I was lying there in a somewhat depressing third-class ward, on a low-budget student insurance policy, thinking about the abstractions of modal logic was my escape. One evening, the crucial uncountable frame and the Löwenheim-Skolem argument that was the core of my first published JSL paper suddenly appeared before my eyes.

My student generation was equally remarkable. These were the days of Liberation in the air, the barriers of rank between students and professors were down, and we all expected a golden new age for the whole planet Earth, which was uniformly inhabited by kind and reasonable people anyway. Looking back, many things that would seem unusual now seemed perfectly normal then. I hitchhiked extensively, starting alone and picking up companions on the way, from Holland to lots of countries, on a minimal budget, including North Africa, Iran, Afghanistan, India, Nepal, and the Soviet Union. My parents wanted to know where I was going, but I told them I did not know my destination exactly, there was nothing for us to discuss anyway, but they could write me poste restante in Tehran, Kabul, or Kathmandu. And it worked: I found letters from my mother waiting for me, and sent terse postcards in return (one has to limit oneself to essentials when communicating with anxious parents). My current academic trips to what are considered exotic countries are very pale copies of these student travels, whose adventures (good and also bad) have formed me for life.

My dissertation topic of modal correspondence theory was also a reflection of my Amsterdam environment. I did not want to do proof theory or intuitionism (these were the old topics my professors did, I wanted to be myself in this new age), but I did want to bridge between the mathematics and the philosophy in my environment. Correspondence theory was a way of employing techniques from classical logic to understand modal logic, then still the paradigm of a philosophically motivated system. My experience in that work continues to determine my general attitudes in research: developing modal and classical logic in tandem, and in the same spirit, being wary of ideological choices between logical systems, but also, appreciating that small languages qua expressive power can be beautiful, and being able to analyze phenomena at different levels of zoom. I think it is such broad themes that define a field, rather than specific formal systems or subfields, and I was happy to see later that creative mathematicians and philosophers of my acquaintance feel the same way. My supervisor Löb was not very supportive in all this, since he disliked modal logic and constantly worried whether it was respectable. Still, I learnt a lot from him in many other ways, and what I did not get from him, I got through the support of Dick de Jongh, and at a crucial moment in writing my dissertation also Anne Troelstra.

Varied personal experiences with research continued of course. Lots of topics in the dissertation revealed their true sense only much later, such as the discovery of bisimulation and proving what is now called Van Benthem's Theorem, then a side comment on the modal language used on models rather than frames. There were also major disappointments, such as having proved the Sahlqvist correspondence theorem independently, but running into an anonymous JSL referee who happened to know that some Norwegian guy had an unpublished thesis with this result. End of the story. I felt an early urge to collect my work into a book, and was invited by a Polish colleague to publish it with Ossolineum around 1979. The book never appeared there, it came out in 1983 as *Modal Logic and Classical Logic* with Bibliopolis in Naples, but not every story needs to be told here. I did get an advance for the book in Poland which was deposited for me in a bank in Warsaw, and annual statements duly arrived. I may have been the only one in my generation to have a capitalist nest egg in a communist country.

After this period, I thought the modal phase should be closed. I looked around for new topics, and for a while, I tried the philosophy of science. I liked some things that I saw, especially the logical analysis of empirical theories, opening my eyes to the fact that there is more to science from a logical perspective than pure mathematics. But I found Sneed's work, the major formal paradigm at that time, largely definitionmongering without very exciting questions, so eventually, I gave up. By the way, interests fade in my life, not because I come to despise their topics or practitioners, but the initial love degenerates into a mild appreciation that is not enough for action.

My next enterprise was the logic of time, where I had become enamored of developing an alternative interval paradigm, rather than points, as primitive entities. At Jaakko Hintikka's invitation, I wrote up my lecture notes into a book The Logic of Time, which brought together structure theory of intervals with techniques from modal logic. This idea was in the air around 1980, and many people proposed it. I remember attending a colloquium by a speaker at Stanford, who was announced as a brilliant leader in Artificial Intelligence having revolutionized our understanding of time, and then telling us something that sounded much like my work. I considered speaking up, but did not: why be the European spoil-sport who points out in bad English that he already had these ideas in an obscure book in some insignificant country? But that evening, I decided to call the speaker in his hotel room, and he said he had just heard my name over dinner. Jon Barwise had told him that people should stop giving talks about temporal logic before they had read van Benthem's book. Sometimes (but do not get your hopes up too much) life deals us sweet surprises. The Logic of Time has been one of my most widely read publications, far beyond the impact of my modal logic, and I have heard back from readers in the most diverse walks of life, from Dutch high school students to Austrian architects. The book is out of fashion now, and major handbook articles on temporal logic do not even mention it. But I am sure that the interval paradigm will make a comeback: it always has, it is just too natural to die. By the way, my later interest in logics of space is a natural

continuation of this work, and in particular, editing the *Handbook of Spatial Logics* in 2007 was a labor of love.

Most of my work in the 1980s was on logic and natural language. This connection was already in the air in my student days. A group of us physics students would go to the faculty of Humanities to take classes in Chomsky's new formal grammar, with the side benefit of being able to watch the gorgeous fashion show at lunchtime when the literary students took their break. Even hardcore scientists have occasional longings for a better, more beautiful life. One day I had learned that the Dutch language has infinitely many sentences, and I rushed to my landlady [I lived in a tiny student room under her wings] to tell her about this wonderful insight. When she heard the trivial proof by recursion on "(Mary thinks that John thinks that)* the weather is bad", she was very disappointed, and told me not to be silly. But the feeling that "there is gold in them there hills", as Austin said about natural language, persisted, and significantly, logicians that I admired such as Hans Kamp and Jon Barwise had moved in that direction. While my initial reaction to Montague grammar had been mainly like that to Sneed: a grand machine with too many definitions and too few real results, things were changing now, and I jumped in.

Topics that intrigued me were not formalizing fragments of natural language, but general themes such as the power of human languages for describing reality, with a focus on their quantifier repertoire. I joined the small band of logicians working on generalized quantifiers, and went for questions of expressive power in definability and semantic universals about shared conceptual structures across natural languages. Eventually, I developed an interest in natural logic of reasoning close to the linguistic surface, resulting in the 'monotonicity calculus', and procedural-computational views of linguistic interpretation, that led to my work on 'semantic automata'. You can find all these themes in my book *Essays in Logical Semantics* of 1986, written toward the end of my period in Groningen, and the informal start of the ILLC in Amsterdam.

Despite what I just said about Montague Grammar, the general machinery behind the complex syntax of natural language did come to intrigue me. I opted for categorial grammar in the elegant version proposed by Lambek in 1958, and really brought to the world's attention in the dissertation of Wojciech Buszkowski. One of my early discoveries turned out to be another disappointment. I found a truly beautiful correspondence between categorical derivations and special linear terms in the lambda calculus, but then learnt that it was a special case of the well-known Curry-Howard isomorphism. All that had been revealed was my ignorance of basic proof theory. I made up for this by entering a proof-theoretic phase concentrating on grammar, recognizing power, and related topics, and developed a wide-ranging theory of language in a categorical perspective, which you can see in my book Language in Action. Categories, Lambdas and Dynamic Logic of 1991. This work also sits at a cusp with the more general idea of resources and substructural rules, as occurring in relevant logic and linear logic, that are still so prominent in logic today. Some of my work even had some practical impact, such as a simple numerical invariant for pruning the search space of Lambek derivations that I once saw running on a TNO computer with a banner streaming on the screen computing successive 'van Benthem counts'. Pure

theorists may brag about the virgin uselessness of their work, but the experience of having an actual use can be very powerful.

Herman Hendriks claims that each of my books has an odd chapter that predicts the next line. The dynamic logic in my title was certainly significant, and I shifted my interest away from natural language. Even so, I did edit the *Handbook of Logic and Language* with Alice ter Meulen in 1997, as a public service to the field at a time when the partisan fights of earlier phases were abating, and the true achievements became visible. By the way, historically, churches and sects have been very successful forms of organization, so I do not want to belittle the power of partisanism.

Moving from language, my interest became the general notion of information. I was struck by the many conceptual parallels in the study of natural language, computer science, AI, and philosophy, and my paper 'Semantic Parallels in Natural Language and Computation' at the 1987 Granada Logic Colloquium contains a host of these, many of which became separate research lines. These include the abstract analysis of intuitionistic and modal information models, substructural characterizations of styles of inference and update, and other things that still occupy me, such as the connection between proof-theoretic combination of pieces of evidence and model-theoretic views of information. These concerns return in the *Handbook of the Philosophy of Information* that I edited with Pieter Adriaans in 2008. Some occur in the editorials, and many more in the chapter on 'Logic and Information' with Maricarmen Martinez, where we try to come to grips with the variety of notions of information in logic, semantic, proof-theoretic, and also correlation- and channel-based as in the situation theory of Jon Barwise, John Perry, and their school.

But information should not be studied on its own. One powerful idea in computer science that has always appealed to me is the dictum of 'no representation without process'. One should know the process a representation is made for, a point that is still underappreciated in natural language semantics and large areas of philosophy. So, along with my interest in information came an interest in computation. By that time, the importance of bisimulation as a view of process equivalence (rediscovered independently in the early 1980s) had become clear to me, and so, around 1990, a return to modal logic made sense. I started doing work on dynamic logics of computation and action in general, and have kept working along these lines, taking my earlier modal work to the area of fixed point logics for induction and recursion.

One aspect of my taking computation seriously was an interest in computational complexity, the mathematics of difficulty of tasks. I have come to believe that complexity is an essential aspect in truly understanding the topics we usually study, and this interest led to a new look at the undecidability or decidability of logical systems. I became interested in the exact reasons for the usual commonplaces such as 'predicate logic is undecidable'. Does this really tell us that core reasoning with quantifiers is complex, or might there be historical accidents of formulation? This is the line that led to the discovery of decidable core logics of relational algebra ('arrow logic') and of predicate logic based on generalized semantics, but also, in another manifestation, the Guarded Fragment of first-order logic, a large decidable realm far beyond basic modal logic. My general feeling is that we should always distinguish between true contents of logical systems and 'wrappings', accidents of set-theoretic

formulations or other fashions. Then there may be much more decidability and even lower complexity in logic than is usually thought. I once gave a talk on my new 'geometric' semantics for predicate logic at the Berkeley logic seminar, and Leon Henkin told me that they still had discussions in the 1950s about what should be the right formulation of first-order semantics. Over dinner, Henkin added that he would have loved to see me debate with Tarski. Well, we shall never know.

All this set the stage for the main theme of this book, that of logical dynamics as an integrated view of the nature of logic as a dual study of statics and dynamics. There is no need for me to repeat this here, since it has been explained in various pieces at the beginning of this book. One of the earliest moments I felt that I was on to something big occurred in 1991 when preparing for an invited lecture on logic and information flow at the Congress of Logic, Methodology and Philosophy of Science in Uppsala. However, I was quickly put back on the ground. On the eve of my lecture, there was a party at Dag Prawitz' house in Stockholm, and I managed to lose my way and miss the last train. There was of course no way I would go back to my distinguished colleague and confess that I could not even remember a few simple travel instructions. So I found a bench at the station and prepared for sleeping out, as I had done so often as a traveling student. All around me were somewhat shady characters, drunks and addicts, but I hung on to my spot. Around 1:30, I suddenly woke from my fitful slumbers: the police were sweeping the station clean, and turning us out into the street, with long subsequent hours of deep cold and discomfort. I found an early morning bus to Uppsala, and gave my talk, but the intellectual epiphany had disappeared.

The progress of my ideas on logical dynamics is easy to follow in books. *Exploring Logical Dynamics* collected many themes and results, with major developments coming out of collaborations with colleagues like Hajnal Andréka and Istvan Németi and Jan Bergstra, a new habit that I acquired in this period, perhaps in line with the logical dynamics idea. Another prominent feature was the work done by my Ph.D. students, who enabled me to see much further than I could have done on my own (perhaps they also did some of the more dangerous missions). You will see many of the themes I mentioned earlier, now as threads in one overarching endeavor.

Conspicuously missing, however, was the theme of multi-agent interaction, which only entered after I became influenced by students like Willem Groeneveld, Jelle Gerbrandy, Hans van Ditmarsch, and (though it is hard to think of him as having been a student) Alexandru Baltag. Dynamic-epistemic logic was born around 2000 (the current ascriptions to Plaza, whose work was totally unknown then, are a form of overblown courtesy that distorts the historical record), and I became an enthusiastic participant. I had a traveling talk called 'Update Delights' in 1999, and still remember an invited lecture at the ESSLLI Summer School in Birmingham where the chair pointed out that my title was a rare instance of a two-word expression in English that is three-way ambiguous. My book *Logical Dynamics of Information and Interaction* from 2011 tells the story as I see it now, with logic as a theory of agency where pure information and knowledge update based on observations, inferences and acts of communication such as questions needs to be in balance with agents' beliefs and how they correct themselves. Much of our quality resides in learning from errors,

and the point is that logic can incorporate this essential feature. In addition, the book reflects another growing conviction of mine, that just dealing with pure information may not be a natural boundary. In all we do, information is in balance with how we evaluate the world, and again logic is up to the task of describing this.

Of course, there are also persistent technical strands from my earlier work in all this, such as the central role of modal logic and dynamic logic, and the use of mathematical notions in logic to demystify mysterious innovations. For instance, the down-to-earth analysis of update as relativization was one of my points at Jelle Gerbrandy's thesis defense. Another point at that defense were connections between dynamic-epistemic logic and Process Algebra which have not panned out yet as I hoped. And in recent years, I have taken up modal frame correspondence analysis of dynamic-epistemic logics, returning to themes and techniques from my dissertation.

My most recent book is Logic in Games, and I see its emphasis on the social process of intelligent interaction (a phrase with a nice Mozartesque ring that I once coined for a strategic European funding program) as a fitting ending to the logical dynamics trilogy. Games in logic had always been on my radar, ever since I read the Luce and Raiffa classic Games and Decisions as a student, and then started out as a young teacher in the 1970s telling my students about Lorenzen dialogues and Hintikka evaluation games. But a deeper interest only started at the time of my Spinoza Award project in 1996, a sort of oeuvre award of the Dutch national science organization that allowed me to pursue new lines by offering a substantial sum of money for 5 years that I was free to spend. I chose three: computational logic, didactic innovation in logic, and logic and games, where we first entered into serious contacts with game theorists, a congenial mathematical community. Incidentally, spending the money turned out to be not totally free. When the award was announced, I had quickly computed that it sufficed for buying one of the smaller Florida keys, and I felt that buying an island for logic in the Caribbean might be the best investment in perpetuity that anyone could make for our field. But that was one step too far for our national science foundation, who refused to think big like our seafaring ancestors.

Logic and games have been a natural match ever since people started thinking about argumentation in Greek and Chinese antiquity, and in my book, I show how dynamic-epistemic logics can analyze the structure of games in innovative ways, leading toward a love child of logic and game theory that might be called a Theory of Play. But I also study the manifold current uses of games to understand logic, and these two themes, 'logic of games' and 'logic as games' form two intertwined strands in my book, which also presents many hybrids between them. I now see this entanglement of strands as the DNA of logic, but how the duality works exactly is still a mystery to me.

What is next? One thing that just seems to be happening naturally these days is a return to philosophy. I feel that the sort of logics I am pursuing now might transform the logic-philosophy interface that has been a bit dormant after the roaring 1960s and 1970s, and one project is a book on epistemology called *The Music of Knowledge* with Alexandru Baltag and Sonja Smets. Another influence that I feel is one that was entirely absent in my student days: the importance of empirical facts about human reasoning, as they are coming to light these days in cognitive (neuro-)science. I feel

that logical theorizing should balance 'intuitions' with reality checks, but facts still scare me a bit, and I am mainly content with admiring those of my current students who seem equally at home in mathematical logic and cognitive psychology. My 2006 paper 'Logic and Psychology: Do the Facts Matter?' shows my cautious, and hence ambiguous, enthusiasm in this realm. One way in which I may face the facts is in a return to my old interests in natural language, where the logical dynamics perspective suggests very different views of what we can, and perhaps should, study by way of key expressions and phenomena. I now believe that the usual emphasis on successful communication is too limited, and that there is much more to the dynamic stability of language with fallible users that has escaped our attention so far. But sometimes, there are even more brute facts than that. Nowadays, I often show students in logic classes frequency tables of words in English or Chinese text corpora, to see which expressions really occur a lot. Fortunately, many logical items score very well.

Finally, I am still intrigued by many technical issues, of which the interface of logic and probability is probably the most urgent right now. Looking at the realities of research in formal philosophy, but also many other fields adjoining logic, this combination seems inevitable, also for deep theoretical reasons. The way I see it now, the mind works on an analogy with the body. Our conscious span of bodily control is in a tiny physical zone, around one meter say, with the bulk behavior of atoms and molecules underneath, and that of astronomical constellations above us. Likewise, our neat little world of conscious deliberation, communication, and decision is just a tiny slice in between the statistics of neural nets in our bodies and the statistics of the crowds and societies of which we form part. I would love to understand these interfaces better, and it may involve deep connections between logic and probability beyond those we already know. To do this well, I may well have to go back to the physics studies of my early student days—something which my sons have been urging me to do anyway while I still have the brain power.

Was logic a good choice? An interviewer of the Dutch national radio once asked me, off the record after a public broadcast on logic, why someone like me had not gone into really interesting subjects like physics or literature instead of this very narrow topic that he found small-minded, being self-centered around our own thinking. But logic has been good for me. It fit with the needs of a young boy who could not choose between the humanities and the sciences, and it put me at an intellectual crossroads between disciplines that keeps opening new vistas, with congenial colleagues at Amsterdam, Stanford, and now also China. And in addition to the delights of research, it sometimes afforded moments of transcendence. I once gave a talk on logic in Ayacucho, high in the Andes, for an audience of mathematicians who only spoke Spanish (and perhaps Quechua), so I talked in Dutch and a friend of mine translated. And my friend told me that at one moment he felt his own personality had disappeared, since the audience was obviously understanding what I was saying through him while he did not. That is the power of resonance afforded by logic.

Of course, there are only few logicians, so I have always tried to work in environments where I would be the average rather than the exception, such as the ILLC in Amsterdam, CSLI at Stanford, or the ESSLLI Summer Schools. Moreover, there have always been enough students sharing my constellation of interests, that circle around colleagues in different fields like electrons, hard to detect at first, but crucial to keeping the whole process working together. In fact, students have been an integral part of my intellectual development, and all my recent books testify to that role. As I said, they enable one to see and achieve much more, as a sort of extended eyes and ears (though not in the sense of the ancient Persian imperial court).

But I should not over-systematize or rationalize my life's choices. In addition to all the rational factors outlined here, I also owe an enormous debt of gratitude to mere chance, or at least circumstances beyond my control. I got my first university job because my professor Löb saw something in me, and I was recalled to Amsterdam in 1986 because my old teachers were willing to take a chance against prevailing currents of thought in mathematical logic. I met many people who influenced me in totally unpredictable productive ways, such as my Dutch high school friend Frans Zwarts or new friends like Jon Barwise or Dov Gabbay. I found highly creative students who chose to study with me though their talents would have taken them anywhere, and their opinions and needs often affected the course of my own work. Sometimes, I think that is all there is to life in general: a beautiful accident.