

String Theory  
Take Home Set 1  
Hand in on March 7, 2007

**Problem 1**

The Virasoro algebra reads

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}.$$

We take so-called "highest-weight state" of the Virasoro algebra, which is by definition a state  $|\phi\rangle$  that obeys  $L_0|\phi\rangle = h|\phi\rangle$  and  $L_m|\phi\rangle = 0$  for  $m > 0$ . The number  $h$  is called the conformal weight. In order to find zero-norm states we can proceed as on page 46 in the book, but we can also do the following: define the two states

$$|a\rangle = L_{-2}|\phi\rangle, \quad |b\rangle = L_{-1}^2|\phi\rangle$$

and form the two-by-two matrix of inner products

$$\Delta = \det \begin{pmatrix} \langle a|a\rangle & \langle a|b\rangle \\ \langle b|a\rangle & \langle b|b\rangle \end{pmatrix}.$$

- i) Find  $\Delta$  as a function of  $c, h$ .
- ii) Take  $h = -1$ . For which value of  $c$  does  $\Delta$  vanish? Did you expect this result? Why?
- iii) Take  $c = 1/2$ . Find the three values of  $h$  for which  $\Delta$  vanishes. These three values play a prominent role in the field-theoretical description of the Ising model at the critical temperature.

**Problem 2**

Do problem 2.7 in the book.

**Problem 3**

Do problem 2.11 in the book.

**Problem 4**

Do the attached extra problem.