# Mathematics from secondary school to university in The Netherlands

Kolloquium Institut für Mathematik

Johannes Gutenberg Universität Mainz, 15.04.2010

Jan van de Craats

Universiteit van Amsterdam

Age 4 - 12: basisschool (Kindergarten + primary school)

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In havo and vwo, in the upper 2/3 years, there are four profiles:

Natuur en techniek (Nature and technics, N & T)

Age 4 - 12: basisschool (Kindergarten + primary school)

Thereafter splitting into three main streams:

- vmbo (4 years), preparing for mbo (2 years or 4 years)
- havo (5 years), preparing for hbo
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- Natuur en techniek (Nature and technics, N & T)
- Natuur en gezondheid (Nature and health, N & G)

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- Economie en maatschappij (Economy and society, E & M)

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- Natuur en gezondheid (Nature and health, N & G)
- Economie en maatschappij (Economy and society, E & M)
- Cultuur en maatschappij (Culture and society, C & M)

Wiskunde C	480 slu
Wiskunde A	520 slu
Wiskunde B	600 slu
Wiskunde D	440 slu

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Wiskunde A	Mathematics light	520 slu
Wiskunde B	Mathematics	600 slu
Wiskunde D	Mathematics extra	440 slu

Four 'flavours' (mathematical packages) for vwo:

Wiskunde C	Mathematics ultra light	480 slu
Wiskunde A	Mathematics light	520 slu
Wiskunde B	Mathematics	600 slu
Wiskunde D	Mathematics extra	440 slu

Wiskunde D is an optional subject. Not required for any university study

Wiskunde D only in combination with Wiskunde B

Wiskunde D only in the profile N & T



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Wiskunde B	600 slu
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Wiskunde A	Mathematics <i>light</i>	520 slu
Wiskunde B	Mathematics	600 slu
Wiskunde D	Mathematics extra	440 slu

Three 'flavours' (mathematical packages) for havo:

Wiskunde A	Mathematics light	520 slu
Wiskunde B	Mathematics	600 slu
Wiskunde D	Mathematics extra	440 slu

Wiskunde D is an optional subject. Not required for any hbo study

Wiskunde D only in combination with Wiskunde B

Wiskunde D only in the profile N & T



#### Profiles and mathematical flavours

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Since 2007, in havo and vwo the following flavours are required:

N & T Wiskunde B (in addition also Wiskunde D (optional))

N & G Wiskunde A (or B)

E & M Wiskunde A (or B)

C & M Wiskunde C (or A or B) (only vwo)

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Recently, many measures have been taken to remediate this situation. Will it be effective? We shall see.

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Citation (Kees van den Hoeven, Free University, Amsterdam):

'Voor eerstejaars studenten is het vaak ook een raadsel hoe het kon gebeuren dat zij op het vwo (binnen Wiskunde A) niet of niet goed zijn voorbereid op hun universitaire studie economie of bedrijfswetenschappen, terwijl ze formeel gesproken aan alle toelatingseisen voldoen.'

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'De grafische rekenmachine wordt kwistig te pas en te onpas gehanteerd, maar welke belangrijke functies achter de knopjes sin en log schuilgaan weet men niet! ("Hoefden we niet te kennen.")'

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'En ondanks deze bijstelling moeten we toch telkens weer aan het eind van het eerste jaar zo'n 25 procent van de eerstejaars een *bindend negatief studieadvies* verstrekken waarbij de gebrekkige wiskunde-voorkennis een belangrijke rol speelt.'

Many didactics people and math teachers in The Netherlands consider Wiskunde A as a big success. It is founded on the ideas of realistic mathematics education (RME), originating at the Freudenthal Institute of Utrecht University. All mathematics is embedded in so-called realistic contexts, and all computations are done with the aid of a graphing calculator (GC)

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- ▶ But the other subjects of wiskunde A are mainly Spielerei with the graphing calculator. Pupils don't acquire any mathematical skills from this. Even the modelling in most instances is poor or even faulty.
- There is no follow-up for wiskunde A in any university study.



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All first year students economy and management science get a crash course algebra and pre-calculus as a preparation to 'real' mathematical courses in the first and second year. Many don't pass these exams and in many cases have to leave university because of a lack of mathematical skills.

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Schotting ended his lament with three firm strokes with a fist hammer on such a *weapon of math destruction*. This resulted in an ovational applause from the audience.



photo: Piet Wesseling (em. hl. TUD), afterwards



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The device looked rather unaffected after this action, but, as Schotting said: 'It didn't function anymore, and that's what matters!'



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- ▶ All that can be done by a GC, can be done much better and with much more insight using *excel* or computer algebra.
- ▶ It impedes mastering 'pre-calculus' (in Wiskunde A and B).
- It is not necessary as a tool in written math exams for testing 'pre-calculus' (in Wiskunde A and B) or analytic geometry (in Wiskunde B)

In which studies mathematics is used?

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In all these studies, in addition to the entrance level in mathematics (wiskunde A or B), many courses in special mathematical subjects are given, aimed at the use of mathematics as a tool in these studies.

Examples of mathematics courses in university studies in science:

▶ linear algebra

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- logic



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Characteristic for all of these 'service subjects':

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- 'Problem solving abilities' and 'creativity' don't play any role.

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- This means that acquiring mathematical skills is all-important.
- For this, mathematical axiomatics and proof techniques are not necessary.

What do *you* think of the following controversial(?) statements concerning pre-university mathematics teaching?:

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- Learning by discovery is the best kind of education (constructivism).
- Learning has to be fun.

### Zum Abschluss:

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 ${\tt http://staff.science.uva.nl/\sim craats}$   ${\tt Vielen\ Dank!}$ 

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